(54) Title: A SYSTEM AND METHOD FOR DETERMINING PRODUCTION PLANS AND FOR PREDICTING INNOVATION

(54) Titre: SYSTEME ET METHODE DE DETERMINATION DE PLANS DE PRODUCTION ET DE PREDICTION D'INNOVATION

(57) Abstract

The present invention relates generally to design and evaluation of research and development, technology transfer, and learning-by-doing, and more particularly to the determination of production plans and the prediction of innovation. A preferred embodiment comprises a method for determining a production plan comprising the steps of: defining a plurality of production recipes such that each of said production recipes is a vector of n operations; selecting a current one of the production recipes; evaluating the current production recipe to determine its cost; modifying the current production recipe to create a trial production recipe (130); evaluating the trial production recipe to determine its cost (140, 150); and assigning the trial production recipe to the current production recipe if the cost of the trial production recipe is less than the cost of the current production recipe.

(57) Abrégé

Cette invention, qui a trait, d'une manière générale, à la conception et à l'évaluation dans les domaines de la recherche-développement, du transfert de technologies et de l'apprentissage par la pratique, concerne, plus précisément, la détermination de plan de production et la prédiction d'innovation. Un mode de réalisation préféré porte sur une méthode de détermination de plan de production consistant à définir plusieurs procédures de production de sorte que chaque procédure de production soit un vecteur de n opérations, à choisir une des procédures en cours, à l'évaluer afin de déterminer son coût, à modifier la procédure de production en cours afin de créer une procédure de production d'essai (130), à évaluer cette dernière afin de déterminer son coût (140, 150) et à affecter cette procédure de production d'essai à la procédure de production en cours si son coût est inférieur à celui de la procédure en cours.

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(57) Abstract

The present invention relates generally to design and evaluation of research and development, technology transfer, and learning-by-doing, and more particularly to the determination of production plans and the prediction of innovation. A preferred embodiment comprises a method for determining a production plan comprising the steps of: defining a plurality of production recipes such that each of said production recipes is a vector of n operations; selecting a current one of the production recipes; evaluating the current production recipe to determine its cost; modifying the current production recipe to create a trial production recipe (130); evaluating the trial production recipe to determine its cost (140, 150); and assigning the trial production recipe to the current production recipe if the cost of the trial production recipe is less than the cost of the current production recipe.

Description

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A SYSTEM AND METHOD FOR DETERMINING PRODUCTION PLANS AND FOR PREDICTING INNOVATION

FIELD OF THE INVENTION

5   The present invention relates generally to design and evaluation of research and development, technology transfer, and learning-by-doing, and more particularly to the determination of production plans and the prediction of innovation.

BACKGROUND OF THE INVENTION

According to the neoclassical model a production plan is merely a point in input-
10 output space. The neoclassical model has been extended to accommodate inter-temporal features such as the variability over time of factory supplies, uncertainty about the production process, and uncertainty about prices. The neoclassical model of production is not, however, fully dynamic since it does not provide a microeconomic basis for explaining technological evolution due, for example, to learning-by-doing, education and training, research and
15 development, or technology transfer.

In this regard, macroeconomics is ahead of its microeconomic foundations. In his celebrated article on learning-by-doing, Arrow (1962) (K.J. Arrow, The economic implications of learning-by-doing, *Review of Economic Studies*, 29:155-73, 1962) accounted for the observed fact that unit production costs could fall even in the absence of capital
20 accumulation and R&D effort. Arrow attributed the increased productivity to learning-by-doing on the shop floor by production workers and managers. Arrow modeled learning-by-doing as a positive macroeconomic *production externality*: increases in "manufacturing experience"—as measured (for example) by cumulative gross investment—lead to increased productivity. Several other macro models of technological progress were based on some
25 production externality. See, for example, Clemhout and Wan (1965) (S. Clemhout and H. Wan Jr., Learning-by-doing and infant industry protection, *Review of Economic Studies*, 32:233-240, 1965), Shell (1967) (K. Shell, A model of inventive activity and capital accumulation, in K. Shell, editor, *Essays on the Theory of Optimal Economic Growth*, pages 67-85, 1967), Stokey (1988) (N.L. Stokey, Learning by doing and the introduction of new
30 goods, *Journal of Political Economy*, 96(4):701-719, 1988); Romer (1990) (P.M. Romer, Endogenous technological change, *Journal of Political Economy*, 98(5):571-5102, 1990);

and Lucas (1993) (R.E. Lucas Jr., Making a miracle, *Econometrica*, 61(2):251-272, March 1993).

Another class of (not unrelated) macro models of technological evolution was based on *non-conventional factors of production*. Uzawa (1965) (H. Uzawa, Optimum technical

5   change in an aggregative model of economic growth, *International Economic Review*, 1965) introduced in a simple growth model *"human capital,"* the stock of which can be increased by devoting resources to education. In the hands of Lucas (1988) (R.E. Lucas Jr., On the mechanics of economic development, *Journal of Monetary Economics*, 22:3-42, 1988); Caballe and Santos (1993) (J. Caballe and M. Santos, On endogenous growth with physical

10   and human capital, *JPE*, 101(6):1042-1067, December 1993) and others, this human capital model (with clearly modeled externalities) became a staple for analyzing productivity growth. Shell (1966) (K. Shell, Toward a theory of inventive activity and capital accumulation, *American Economic Review*, 56(2):62-68, May 1966); Shell (1967); Shell (1973) (K. Shell, Inventive Activity, industrial organization and economic growth, in J.A. Mirrlees and N.H.

15   Stern, editors, *Models of Economic Growth*, pages 77-100, 1973); Romer (1986) (P. M. Romer, Increasing returns and long-run growth, *Journal of Political Economy*, 94:1002-1037, 1986); and Romer (1990) introduced into growth models the macro variable *"technological knowledge."* The Shell-Romer model combines "technological knowledge" (or "stock of patents") with production externalities and increasing returns-to-scale to analyze

20   the role of industrial organization in growth, the dependence of growth on initial conditions, and other important macro problems.

Macroeconomic models based on production externalities and/or non-conventional inputs have been useful in raising important issues about public policy toward technology and in explaining observed increases in aggregate output, but the inadequacy of the

25   microeconomic foundations of these models is a serious problem for the analysis of production.

The neoclassical economic model of production (see e.g., Debreu (1959, Chapter 3) (G. Debreu, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, 1959), Arrow and Hahn (1971, Chapter 3) (K.J. Arrow and F. Hahn, *General Competitive Analysis*,

30   1971), and the references to the literature therein) is a reduced-form model of *existing* technological possibilities. Each firm is endowed with a *technology set*—a set of technologically

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feasible input-output combinations. These technology sets are assumed to be fixed parameters of the neoclassical economy.

A simple example with one input and one output is described in **FIG. 16**. A neoclassical production plan is a point (x, y) where $x \geq 0$ is the quantity of the input and

5   where $y \geq 0$ is the quantity of the output. The (shaded) set T is the set of all feasible neoclassical production plans. The northwest boundary of T is the production possibility frontier (PPF). Points on the PPF represent the "efficient" production plans according to the neoclassical model, since no other plan can be found that gives either more output for the same input or less input for the same output. In this example, the PPF is linear, i.e.,

10 production exhibits constant returns to scale. The production function is

$$y = \theta\, x,$$

where the positive scalar θ is the slope of the PPF.

From the viewpoint of neoclassical production models, production plan A in **FIG. 16** is "efficient." Production plan B is "inefficient." In fact, A strictly "dominates" B: pair A

15 yields more output with less input than pair B. A and B are nearby (or similar) production plans. C is distant from A and B.

Now look at **FIG. 16** from the engineering point of view. Suppose that engineers tell us that there are only two known processes (Recipe 1 and Recipe 2) for producing this output. Recipe 1 supports all pairs $(x, y) \geq 0$ that satisfy $y \leq \theta x$, where θ is a positive scalar. Recipe 2

20 supports all pairs $(x, y) \geq 0$ that satisfy $y \leq \theta' x$, where $\theta' < \theta$ is a positive scalar. The production pairs A and C lie on the ray $y = \theta x$, while pair B lies on the ray $y = \theta' x$ (indicated in **FIG. 16** by the dashed line).

Suppose Recipe 1 is from an engineering viewpoint very different from Recipe 2, even though pair A is close to pair B -- would it ever be rational to produce at B? The answer

25 is yes. Suppose that Recipe 2 is relatively untried. Using Recipe 2 might lead to the discovery of recipes close to 2, but with lower production costs than Recipe 1 (and Recipe 2). A case could be made in some circumstances for using only Recipe 2 and in other circumstances for using both Recipe 1 and Recipe 2 simultaneously.

According to the neoclassical model, production pairs A and B are close (or similar),

30 while the production pairs A and C are far apart (or dissimilar). But as measured by their recipes, A and B are by assumption far apart (or dissimilar) if Recipe 2 is used for B, while

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plans *A* and *C* must be based on precisely the same recipe, Recipe 1. Furthermore, the neoclassical model does not accurately represent the opportunities facing the production manager.. He must *jointly* choose the recipe and the production plan. Having chosen Recipe 1, only a production pair from the PPF should be chosen. Having chosen Recipe 2,

5 only production pairs from the dashed ray in FIG. 16 should be chosen. All other pairs in *T* (i.e., those not satisfying $y = \theta x$ or $y = \theta'x$) result from waste of some output or some input when using one or both of the two basic engineering processes.

SUMMARY OF THE INVENTION

10   A preferred embodiment comprises a method for determining a production plan comprising the steps of: defining a plurality of production recipes such that each of said production recipes is a vector of *n* operations; selecting a current one of said production recipes; evaluating said current production recipe to determine its cost; modifying said current production recipe to create a trial production recipe; evaluating said trial production

15 recipe to determine its cost; and assigning said trial production recipe to said current production recipe if said cost of said trial production recipe is less than said cost of said current production recipe.

     A second preferred embodiment comprises a method for predicting technological innovation comprising the steps of:

20   defining a model comprising:

               a plurality of production recipes such that each of said production recipes is a vector of *n* operations; and

               a plurality of model parameters; and

          executing said model comprising the steps of:

25   selecting a current production recipe;

               evaluating said current production recipe to determine its cost;

               modifying said current production recipe to create a trail production recipe;

          and

               assigning said trial production recipe to said current production recipe if said

30 cost of said trial production recipe is less than said cost of said current production recipe.

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BRIEF DESCRIPTION OF THE DRAWINGS

The following detailed description, given by way of example, will best be understood in conjunction with the accompanying drawings in which:

FIG. 1 is a flowchart diagram illustrating the general steps of a preferred embodiment.

FIG. 2 illustrates an analytical representation of a production recipes model used in a preferred embodiment.

FIG. 3 illustrates the δ-neighborhoods used in a preferred embodiment.

FIG. 4 illustrates a technological graph.

FIG. 5 illustrates the analytical representation of cost reduction dynamics used in a preferred embodiment.

FIG. 6 is a table (Table 1) of estimated progress ratios for various industries.

FIG. 7 is a table (Table 2) summarizing the parameters used in a preferred embodiment.

FIG. 8 is a table (Table 3) summarizing the results of a 20-run per parameter vector experiment run on a focused parameter set.

FIG. 9 is a table (Table 4) summarizing the results of a 20-run per parameter vector experiment on a random parameter set.

FIG. 10 is a three part table (Table 5) illustrating the prediction of the sample mean of the estimated progress ratios by OLS scoring from the focused parameter set.

FIG. 11 is a three part table (Table 6) illustrating the prediction of the sample mean of the estimated progress ratios by OLS scoring from the random parameter set.

FIG. 12 is a three part table (Table 7) illustrating the prediction of the sample standard deviation of the estimated progress ratios by OLS scoring from the focused parameter set.

FIG. 13 is a three part table (Table 8) illustrating the prediction of the sample standard deviation of the estimated progress ratios by OLS scoring from the random parameter set.

FIG. 14 is a three part table (Table 9) illustrating the prediction of the mean improvement percentage per measured batch by OLS scoring from the random parameter set.

FIG. 15 is a three part table (Table 10) illustrating the prediction of the mean curvature mis-specification by OLS scoring from the random parameter set.

FIG. 16 illustrates weaknesses of neoclassical production theory.

FIG. 17 is a histogram of estimated firm progress ratios.

FIG. 18 illustrates the phenomenon of plateauing.

FIG. 19 illustrates the SFS (Slow Fast Slow) Mis-specification.

FIG. 20 is a graph of a base parameter experience curve, after one production run.

FIG. 21 is a graph of a smooth landscape, after one production run.

FIG. 22 is a graph of a rugged landscape, after one production run.

FIG. 23 is a graph of a landscape with $n = 10$, $e = 1$, and $s = 100$, after one production run.

FIG. 24 is a graph of the base parameter experience curve, with the same parameters as in the graph in FIG. 20, but averaged over twenty production runs.

FIG. 25 is a graph of a smooth landscape, with the same parameters as in the graph in FIG. 21, but averaged over twenty production runs.

FIG. 26 is a graph of a rugged landscape, with the same parameters as in the graph in FIG. 22, but averaged over twenty production runs.

FIG. 27 is a graph of a rugged landscape, with the same parameters as in the graphs in FIGS. 22 and 26, but averaged over fifty production runs.

FIG. 28 is a graph of a landscape with the same parameters as in the graph in FIG. 23, but averaged over twenty production runs.

FIG. 29 illustrates a way in which parameters are chosen in the preferred embodiment.

FIG. 30 illustrates effects of varying n, the number of operations in the recipe, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p).

FIG. 31 illustrates effects of varying s, the number of possible states for each operation in the recipe, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p).

FIG. 32 illustrates effects of varying e, the number of other operations that affect the cost of each operation, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p) (in the case $s = 2$).

FIG. 33 illustrates effects of varying e, the number of other operations that affect the cost of each operation, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p) (in the case $s = 10$).

FIG. 34 illustrates effects of varying δ, the maximum number of operations changed

5   per trial, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p).

FIG. 35 illustrates effects of varying T, the number of trials, on the sample mean of the estimated progress ratios ($\bar{p}$) and the sample standard deviation of the estimated progress ratios (s_p).

10   FIG. 36 illustrates plateauing effects of varying n, the number of operations, on the sample mean of the estimated progress ratios ($\bar{p}$) and the standard deviation in the percent of trials that result in improved productivity (s_z).

FIG. 37 illustrates plateauing effects of varying s, the number of possible states for each operation in the recipe, on the mean percentage of trials that result in improved

15   productivity ($\bar{z}$), the standard deviation in the percent of trials that result in improved productivity (s_z), and other parameters.

FIG. 38 illustrates plateauing effects of varying e, the number of other operations that affect the cost of each operation, on the mean percentage of trials that result in improved productivity ($\bar{z}$) and the standard deviation in the percent of trials that result in improved

20   productivity (s_z).

FIG. 39 illustrates plateauing effects of varying δ, the maximum number of operations changed per trial, on the mean percentage of trials that result in improved productivity ($\bar{z}$) and the standard deviation in the percent of trials that result in improved productivity (s_z).

25   FIG. 40 discloses a representative computer system 4010 in conjunction with which the embodiments of the present invention may be implemented.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENT

A preferred embodiment is a system and method for modeling technological

30   innovation. One of the more important aspects of a preferred embodiment is in the description of the production plan. To the usual input-output specification is added a

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description of the underlying engineering recipe employed. Describing how one recipe is related to another then allows one to build models that suggest which types of technologies are likely to be uncovered in the course of ordinary shop-floor operations (learning-by-doing), which R&D programs are most likely to be successful, and which types of technologies are 5 ripe for transfer from one firm (or economy) to another.

In a preferred embodiment, a production recipe is described by a vector of basic production operations such as heating, mixing, stirring, shaping, boxing, internal factory . transportation, and so forth. For given outputs, the input requirements for each of the operations depends on the instruction (or setting) given for that operation and the instructions 10 given for some of the other operations. Hence the embodiment allows for production externalities *within* the firm.

A specific application of the (more general) production recipes approach (along with the nascent technology approach) of the preferred embodiment is the construction of a model of shop-floor learning-by-doing. It is assumed that the firm employs a single input to produce 15 a single output and that, for a given fixed recipe, this process entails constant returns to scale. It is also assumed that the firm's output stream is predetermined. The method allows for deviations from the currently reigning technology, but assumes that such *production trials* (or "errors") are not directly controlled by the firm. It is assumed that a newly discovered recipe is either accepted or rejected merely on the basis of current cost efficiency relative to that of 20 the reigning technology.

By correctly choosing the basic parameters of the model, one is able to match the basic statistics and important qualitative phenomena from observed experience curves—including the mean progress ratios (an inverse measure of the slope of the experience curve) and their standard deviations, plateauing (runs without improvements), 25 curvature bias, and sensitivity to the length of the production run. The disclosed method of modeling of recipes works for one input, one output, constant returns cases for which the focus is either the *short term* or the *intermediate term*.

A preferred embodiment comprises the general steps of applying the machinery of the production recipes model to a specific problem (in a preferred embodiment, the learning-by- 30 doing problem), refining the parameters of the model to fit the details of the problem, and then utilizing the model to predict which nascent technologies are the most promising. The

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subject disclosure describes a system and method for constructing a microeconomic model of technological evolution. To the existing (or "currently available") technologies of the neoclassical production model, nascent technologies are added, which include both undiscovered technologies and forgotten technologies. One might be skeptical about any

5   modeling of undiscovered technologies. While existing technologies can be verified by current engineering practice, undiscovered technologies cannot. On the other hand, practicing production engineers and business managers are not reluctant to base important . business decisions on forecasts of technological progress in the firm's manufacturing operations. In fact, one of the most reliable analytic tools in production management is the

10   engineering experience curve (or "learning" curve), which projects existing unit production costs for a given product into its future unit production costs. Among production engineers, marketing managers, business executives, and even corporate directors, empirical learning curves are far better known and more frequently used than are empirical production functions or empirical cost functions.

15   A production recipe ω is a complete list of engineering instructions for producing given outputs from given inputs. In the preferred embodiment it is assumed that the firm uses a single input to produce a single output. It is also assumed that, given the recipe choice, there is no waste in production. It is assumed that in production run t, the firm produces $y_t \geq$ 0 units (production engineers often use a precise (but different for each output class) unit of

20   measurement called the "batch") of output by employing $\eta_t > 0$ units of input (hereafter "labor") based on the recipe ω_t. It is also assumed that there is no uncertainty about the production process. Let $\ell_t = y/\eta_t$ be unit labor cost. Then $\ell_t = \ell(y_t; \omega_t)$, where, for fixed ω_t, ℓ is the average cost function. If ℓ first falls and then rises as y_t is increased, then the average cost curve is U-shaped. If ℓ is independent of y_t, then there are constant returns to scale.

25   The method of representing technologies utilized in a preferred embodiment is an improvement on the approach taken in several methods of modeling technological innovation in which there are two types of technologies, advanced and backward (and two types of firms, also advanced and backward) (see, e.g., Shell (1973) and Grossman and Helpman (1991) (G.M. Grossman and E. Helpman, Innovation and Growth in the Global Economy, 1991)).

30   Advanced firms have access to both the advanced and backward technologies, but backward firms are restricted to the backward technology. Let ω_a be the advanced recipe and ω_b be the

backward recipe. In this literature, the strong non-crossing (in the absence of non-crossing, there might not be a most advanced (or a most backward) recipe) assumption is made, so

$$0 < \ell(y; \omega_a) < \ell(y; \omega_b) < \infty \text{ for each } y > 0.$$

For the advanced firm, the set of recipes Ω_a is given by

5                              $\Omega_a = \{\omega_a, \omega_b\}.$

For the backward firm, the set of recipes, Ω_b, is given by

$$\Omega_b = \{\omega_b\}.$$

In the subject specification, the above very simple model is generalized to allow the firm to choose not merely from (at most) two recipes, but instead from a large general set of

10 recipes, Ω. The formal model in what follows is restricted to constant returns to scale, so that the unit labor requirement depends only on the recipe employed. The labor requirement for a given recipe is not typically known with certainty. Instead there is associated with each recipe ω a probability measure over the set of labor requirements. Although it is also assumed that there is a (relatively small) subset of currently available recipes $\Omega_{CA} \subset \Omega$, and

15 that the respective labor requirements for each of these recipes is known with certainty, this assumption is not critical.

In the preferred embodiment, a production recipe is described by a vector of basic *production operations* such as heating, mixing, stirring, shaping, boxing, internal factory transportation, and so forth. For given outputs, the input requirements for each of the

20 operations depends on the instruction (or setting) given for that operation and the instructions given for some of the other operations. Hence one allows for production externalities *within* the firm. These *intra-firm* production externalities are crucial to the analysis.

The method of a preferred embodiment is to use the production recipes model to provide a microeconomic model for observed learning-by-doing in production, and then use

25 computer simulations to discover which nascent technologies are the most promising. A complete model of production is specified (the IRF) (which includes inputs, outputs, and recipes; as well as current technologies and nascent technologies) and then "closed" so as to model shop floor productivity improvements and hence to explain the observed empirical features of the firm's experience curve. There are three reasons for doing this: (1) The

30 learning-by-doing model is important in its own right for economic theory and economic policy. It would be worthwhile to understand the micro sources of the productivity increases

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and what promotes them, rather than merely representing this process as a fixed macroeconomic externality. (2) Empirical experience-curve analysis is central to management science and management practice. It would be desirable if these experience curves could be explained in terms of basic micro economics. (3) The method of production

5 recipes (and nascent technologies) is quite general, with possible applications for modeling R&D, basic research, and technology transfer. In the disclosed preferred embodiment, the general method is illustrated by a concrete application. There are three reasons why learning-by-doing is a good candidate for the preferred embodiment: (1) Empirical studies of engineering experience curves are abundant. (2) Learning-by-doing permits one to be

10 relatively less sophisticated in modeling the purposiveness of the economic agents, so that one can focus—for the time being—on the relatively sophisticated model of technology. (3) The one input/one output learning-by-doing model allows one to use —with only minor modifications—a tested computer simulation program.

In the production recipes/nascent technology approach of a preferred embodiment, the

15 construction of a model of shop-floor learning-by-doing to determine promising nascent technologies, it is assumed that the firm employs a single input to produce a single output and that, for a given fixed recipe, this process entails constant returns to scale. It is also assumed that the firm's output stream is predetermined. The method allows for deviations from the currently reigning technology, but assumes that such *production trials* (or "errors") are not

20 directly controlled by the firm. It is assumed that a newly discovered recipe is either accepted or rejected merely on the basis of current cost efficiency relative to that of the reigning technology.

These strong assumptions allow the employment of a variant of Kauffman's NK model (see Kauffman and Levin (1987) (S. Kauffman and S. Levin, Toward a general theory

25 of adaptive walks on rugged landscapes, *Journal of Theoretical Biology*, 1987) and Kauffman (1988, 1993) (S. Kauffman, The evolution of economic webs, in P.W. Anderson, K.J. Arrow, and D. Pines, editors, *The Economy as an Evolving Complex System*, 1988; S. Kauffman, *Origins of Order: Self-Organization and Selection in Evolution*, 1993)) to analyze the dynamics of manufacturing costs. The NK model was originally designed for an-

30 alyzing asexual biological evolution. In the evolutionary biology interpretation, it is assumed that the "fitness" of a creature can be represented by a scalar. The corresponding assumptions

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for learning-by-doing are the single output, the single input, and constant returns to scale, which together allow the scalar "fitness" to be replaced by the scalar "current technological efficiency" (the inverse of current unit production cost). In the biological interpretation, it is assumed that genetic changes occur at random and that fitter creatures immediately replace

5 those that are less fit. In the present interpretation, the corresponding assumptions are that shop-floor trials take place at random and that the reigning recipe is replaced by the new recipe if and only if the new recipe is more efficient in the short run – i.e., recipe selection is. myopic.

The preferred embodiment is not restricted to myopic recipe selection. It would

10 certainly be routine for one skilled in the art to extend the model of learning-by-doing to allow for both foresight in selection of the reigning recipe(s) and some control by the firm of the rate and direction of experimentation. For the latter, there must be costs of experimenting. These would include the output losses from pilot-project retrogressions, the opportunity costs of sampling other recipes, and additional resource costs of experimenting

15 with distant recipes.

A preferred embodiment works for one input, one output, constant return cases for which the focus is either the *short term* or the *intermediate term*.

FIG. 1 is a flow diagram illustrating the method of a preferred embodiment. At step

100 a set of one or more production recipes, each recipe comprised of n operations, is

20 represented as a set of recipe vectors $\{\omega\}$ in $\mathbf{R}^n$. At step 110 a technological graph Γ is constructed whose nodes are the recipe vectors ω. At step 120 a measure of distance between the recipe vectors is defined. At step 130 a production recipe trial on Γ is defined to be a recipe vector within a distance $\delta \in \{1, 2, ..., n\}$ of a currently adopted recipe vector. At step 140 productivity gain or loss is defined in terms of an estimated progress ratio, and at step

25 150 production recipes are evaluated by measuring the productivity gain or loss of trials on Γ.

In a preferred embodiment, production is assumed to involve n distinct engineering *operations*. The recipe ω 210 can then be represented by

$$\omega = (\omega^1, ..., \omega^i, ..., \omega^n) \in \mathbf{R}^n$$

where ω^i represents the *instructions* for operation i for $i = 1, ..., n$. See **FIG. 2**. It is assumed

30 that for each operation i the set of possible instructions is discrete. These choices may be "qualitative" (e.g., whether to use a conveyor belt or a fork-lift truck for internal transport) or

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they may be "quantitative" (e.g., the *setting* of the temperature or other knob on a machine). In the latter case, the variable being adjusted is approximated by discrete settings (think of the knob "clicking" from one setting to another). In particular, it is assumed that ω^i is an integer which satisfies

5 $$\omega^i \in \{1,...,s\}$$

for $i = 1, \ldots, n$, where s 220 is a positive integer. Hence the number of recipes (we shall sometimes refer to the recipe vectors and the recipes themselves interchangeably, since context will clarify whether actual recipes or vectors in $\mathbb{R}^n$ are being referred to) is finite and given by

10 $$\#\Omega = s^n.$$

The finiteness of the set $\Omega = \{$recipe vectors $\omega\}$ has serious consequences. A model based on a finite space of recipes does not permit long-run productivity growth. The model with a finite set of recipes is, however, quite appropriate for modeling the intermediate term productivity improvements observed in the manufacture of specific goods (such as a

15 particular model of a microprocessor) over their relatively short product lives (measured in months, years or—at the very most—decades).

It is assumed that the unit labor cost of operation i, $\phi^i(\omega)$, is a random variable whose distribution function is defined on $\mathbb{R}_+$. Consider two distinct recipes, ω and ω'. The random variables $\phi^i(\omega)$ and $\phi^i(\omega')$ are not necessarily independent. In fact, ϕ^i depends on the

20 instructions, ω^i, for operation i and possibly on (some of) the instructions for the other operations, ω^{-i}. (With minor abuse of notation, one could then have denoted the unit labor costs of operation i by $\phi(\omega^i; \omega^{-i})$, or more simply, $\phi^i(\omega)$.) The labor requirements are assumed to be additive; hence

25 $$\phi(\omega) = \sum_{i=1}^{n} \phi^i(\omega)$$

where $\phi(\omega)$ is the unit cost of production employing recipe ω. For ω fixed, $\phi(\omega)$ is a random variable. If ω is allowed to vary over Ω, then $\phi(\omega)$ is a *random field*. A random field is a slight generalization of a stochastic process to allow the argument (in this case ω) to be a vector (as opposed to being a

30 scalar such as "time"). For the special case in which $n = 1$, $\phi(\omega)$ is then an ordinary stochastic process. Denote by $t^i(\omega)$ the realization of the random variable $\phi^i(\omega)$. The

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realization of the random variable $\phi(\omega)$ is $\ell(\omega) = \sum_{i=1}^{n} \ell^i(\omega)$. If ω varies over Ω, the family of

realizations $\ell(\omega)$ is called the *landscape* (of the random field $\phi(\omega)$).

There are six basic parameters in the disclosed model of shop-floor learning-by-doing:
(1) n, the number of production operations, (2) s, the number of possible instructions or

5   settings for each operation, (3) e, the externality parameter that gives the number of
operations whose settings affect the costs of one operation, (4) δ, the maximum step size per
trial, (5) τ, the number of trials made on the shop floor per measured batch, and (6) T, the
length of the production run.

The above discussion shows how the neoclassical notion of technological distance can

10 be misleading. One needs a measure of distance that captures the similarity or dissimilarities
of the inherent production processes rather than the relative efficiencies of production pairs or
their relative scales. "Distance" between recipes will, of course, depend on the application.
If one takes a shop-floor perspective — as is done in the learning-by-doing embodiment of
the method — then ω is near 'ω if these recipes are the same except for, say, one temperature

15 setting. If moving from ω to ω' represents the substitution of fluorine for chlorine where
fluorine was not formerly in use, then one would probably think of ω and ω' as very far apart
in the shop-floor metric. But for an R&D problem, the appropriate metric might be altogether
different. In the chemistry research lab, for example, the distance would in this case be small,
since every chemist is aware that the elements chlorine and fluorine are likely to react

20 similarly because they are from the same column of the Periodic Table of the Elements.

The preferred embodiment assumes that the set Ω can be described so that distances
are meaningful from the appropriate technological perspective. A formal definition follows.

Definition (Distance): The distance $d(\omega,\omega')$ between the recipes ω and ω' is the
minimum number of operations which must be changed in order to convert ω to ω'. Since

25 changing operations is symmetric, $d(\omega,\omega') = d(\omega', \omega)$. Example: Assume that ω and ω'
differ only in the ith component. Then $d(\omega,\omega') = 1$ when $\omega^i = 1$ and $(\omega')^i = 2$ or when $\omega^i = 1$
and $(\omega^i) = 37$.

This definition of distance makes the most sense when the instructions are merely
qualitative. If instead the instructions can be represented by ordinal settings (such as

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temperature), then the distance notion should be different. If the instructions for operation i in the above example had been temperature settings, then the recipe with its ith entry equal to 2 would be closer to ω than the recipe with its ith entry equal to 37. In particular, 2°C is closer to 1°C than is 37°C. If settings are ordered, then a wise strategy for the firm might be

5 to change if possible the setting in the same direction that led to the most recent improvement. If 2°C is an improvement over 1°C, perhaps the next trial should be 3°C. Introduction of ordinal settings and more complicated distance measures is also within the claimed scope of the preferred embodiment. In general, the requirement is that the set Ω have some reasonable notion of distance imposed on it.

10   Definition (Neighbors): Let $N_i(\omega)$ be the set of i-neighbors of recipe ω:

$$N_i(\omega) = \{\omega' \in \Omega | d(\omega,\omega') = i\},$$

where i is a positive integer.

Let $\tilde{N}_\delta(\omega) = \{\omega' \in \Omega | \omega' \in \cup_{i=1,...,\delta} N_i(\omega)\}$

be the set of recipes at least distance one from ω but not more than distance $\delta \in \{1, ..., n\}$

15 from ω. See FIG. 3. $\tilde{N}_\delta(\omega)$ 310 is a δ-neighborhood of ω 210.

With the above definition of distance between recipes, it is straightforward to construct the *technological graph* Γ (see FIG. 4). The nodes or vertices 410 of Γ are the recipes $\omega \in \Omega$. The edges 420 of Γ connect a given recipe to recipes distance 1 away (i.e., to the elements of $N_1(\omega)$).

20   Next comes the definition of the input requirement field, which is a full description of all the basic technological possibilities facing the firm.

Definition (IRF): The *input requirement field* (IRF) is the random field $\phi(\omega)$ (defined over the vertices of the technological graph Γ).

In order to obtain concrete results, one must further specify the IRF. In the next

25 description, the relationship between the random variables $\phi(\omega)$ and $\phi(\omega')$, which in general are not independent, is specified. After that is specified the functional forms for the $\phi'(\omega)$. Except for some of the underlying stochastic structure, this specifies the IRF, but it does not "close the model." To do that, one needs a complete algorithm of which recipes are chosen for production, what is learned by the firm from its experience, and how much of this is

30 remembered.

15

It is assumed that the costs of a given operation depend on the chosen setting for that operation and possibly on the settings for some (but not necessarily all) of the other operations. Define the connectivity indicator e_j^i by

$$e_j^i = \begin{cases} 1 & \text{if the choice of setting for operation } i \text{ affects} \\ & \qquad \text{the labor requirement for operation } j \\ 0 & \text{otherwise} \end{cases}$$

for $i, j = 1, ..., n$. Since the choice of the setting for the ith operation always affects the costs

5 for the ith operation,

$$e_i^i = 1$$

for $i = 1, ..., n$. The number e^i of operations with costs affected by operation i is given by

$$e^i = \sum_{j=1}^{n} e_j^i$$

10 for $i = 1, ..., n$, while the number e_i of

operations that affect the costs of operation i is given by

$$e_i = \sum_{j=1}^{n} e_i^j$$

for $i = 1, ..., n$. Define the set E_i, the

15 set of operations cost-relevant to operation i, by

$$E_i = \{ j \in \{1, ..., n\} \,|\, e_i^j = 1 \}$$

for $i = 1, ..., n$.

The simplifying assumption is made that each operation is cost-affected by $(e$-1) other operations, so that

20                                    $\#E_i = e_i = e$

for $i = 1, ..., n$, where $e \in \{1,..., n\}$. Under this assumption, the labor requirement of any given operation is affected by the settings for that operation and the settings for exactly $(e$-1) other operations. Therefore, there are exactly s^e permutations of the settings that affect the costs of operation i. Each of these is a *sub-recipe cost-relevant to operation i*. Let $\{i_1, ..., i_e\}$

25 denote the elements in the set $E_i = \{ j \,|\, e_i^j = 1 \}$ for $i = 1, ..., n$. Then denote by $(\omega^{i_1}, ..., \omega^{i_e})$ a sub-recipe cost-relevant to operation i (where for convenience i_1 is defined to be equal to i) for $i = 1, ..., n$ (due to typographic difficulties, the symbol $i1$, e.g., should be identified throughout with the symbol i_1). The set of sub-recipes cost-relevant to operation i is a

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projection of the s^r recipes into s^c sub-recipes. There are ns^c such sub-recipes in all. The stochastic unit labor requirement for operation i based on the sub-recipe $(\omega^{i1},...\omega^{ic})$ can be written as

$$\phi^i(\omega^{i1},..., \omega^{ic}), \text{ or } \phi^i(\omega^{ic})$$

5 for $i = 1,..., n$, with only slight abuse of notation.

The parameter e plays a crucial role in the preferred embodiment. If $e = 1$, there are no (intra-firm) external effects among the operations. Each of the operations could as well have taken place in separate firms, since in this case there can be no gains from coordination. With $e = 1$, one would also expect the two random variables $\phi(\omega)$ and $\phi(\omega')$ to be highly

10 correlated if ω' is close to ω, since by definition ω' and ω would have many instructions in common and hence $\phi^i(\omega) = \phi^i(\omega')$ for most i. The larger the parameter e, the less correlation one would be expect between $\phi(\omega')$ and $\phi(\omega)$ even for ω' close to ω. This is because the change in the instructions for one operation affects the costs of several other operations. Hence e is an inverse measure of the correlation between $\phi(\omega')$ and $\phi(\omega)$ for ω' close to ω.

15 The corresponding landscape $\ell(\omega)$ (a realization of the IRF) is typically "correlated," or "smooth," for small values of e, while $\ell(\omega)$ is typically "uncorrelated," or "rugged," for large values of e.

This concludes the disclosure of the general production recipes model; next the specification illustrates the preferred embodiment by applying the general model to the

20 problem of modeling learning-by-doing.

It is assumed that shop floor workers follow the recipe (or blueprint) provided by management, but from time to time, they make small modifications in the current recipe. These modifications are referred to as "trials" in the subject specification. Depending on the context or the interpretation, the trials may also be thought of as "errors" or "informal

25 experiments."

A trial occurs when (1) at least one operation i in the production recipe ω is modified and (2) that modification is observed and is evaluated by the firm (perhaps by the quality-control engineers). It is assumed that modifications occur during the production of a batch, and that observation and evaluation occur when the production of the batch is completed. It

30 is further assumed that there is only one trial per "quality-control batch." This assumption is

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not restrictive: since the *quality-control batch size*, *B*, may or may not equal the *measured batch size*, **B** , used in data-gathering, the *batch deflator*, τ, is introduced. τ satisfies

$$\tau = \mathbf{B}/B$$

where *B* is defined so that exactly one trial is made during the production of the *quality-*

5 *control batch*. Hence if τ is a positive integer it is interpreted as the number of trials per measured batch. In general, $\tau > 0$ is interpreted as the average number of trials per measured batch. Note that $\tau \geq 1$ if **B** $\geq B$, and $\tau < 1$ if **B** $< B$.

One would expect τ to depend on the particular type of manufactured good. The trials parameter τ is likely to be higher for airframes than for computer chips, because the number

10 of operations, n, is likely to be higher in the airframe case. The trials parameter, τ, also depends on management practices, corporate culture, and worker psychology. In a tight production environment, there would be fewer defects, but fewer trials. In a looser production environment, there would be more defects, but also more trials. Hence management is likely to be keen to attempt to control τ if possible to achieve an optimal

15 balance between "sloppiness" and "inventiveness."

A trial can be interpreted in at least three (non-exclusive) ways. The first interpretation is that the trial is a small scale experiment in production to which the firm does not fully commit. This would be a model of R&D in which the only cost of the R&D activity is the missed opportunity for investigating alternative recipes during the period in question.

20   A natural alternative is to assume that the firm must *commit* to the new production recipe in order to sample it: If recipe ω_t is chosen during the production run *t*, then the labor requirement for run *t* will be the realization of $\phi(\omega_t)$. In this case, unit costs may actually increase from one period to the next, i.e., cost retrogressions could occur.

A third interpretation is possible. One can think of the firm operating a large number

25 of distinct production subunits in parallel. Each sub-unit begins a production run at time *t* with an assumed labor requirement $\ell(\omega_{t-1})$, the unit cost for run *t* - 1. During the course of production run *t*, a trial in one production sub-unit leads to production by its subunit using recipe ω'. The associated labor requirement *for this production sub-unit* is then $\ell(\omega')$, the realization of $\phi(\omega')$. However, because there are many production sub-units, the average per-

30 batch cost of production is close to $\ell(\omega_{t-1})$, the unit cost of the (pre-trial) reigning recipe. Consequently, in this scenario, one can think of firms trying new recipes, without substantial

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sacrifices in current labor requirements. This third interpretation is utilized in the preferred embodiment, because it allows simulations to be based on an existing, tested computer program.

The next description is of the trial dynamics and the memory of the firm.

5   Production trials occur on the shop floor level at the rate of 1 per quality control batch B (or τ per measured batch B). It is assumed that the trial recipe is at distance of at least one but *no greater than* $\delta \in \{1, 2,..., n\}$ from the currently adopted recipe ω. It is assumed that the probability of a trial is uniform over the neighborhood $\tilde{N}_\delta(\omega)$. (See **FIG. 3** for an illustration of a neighborhood $\tilde{N}_\delta(\omega)$ where $\omega = (1,3)$, $s = 4$, $n = 2$, and $\delta = 2$.) The

10   probability distribution of trials could be different. For example, a scheme that loads more probability on the recipes closer to ω_{t-1} would in general be more realistic. Such a modification would be a matter of weighting the probability distribution as function of the distance from ω_{t-1}, would be obvious to one of ordinary skill in the art. The quantitative effect of this change would be to reduce the big-step effect for any given δ, but the qualitative

15   effects would not be altered. That is, the extent of modification of existing recipes is limited to recipes that differ from the currently prevailing recipe by no more than $\tau\delta$ operations—the number of trials per production run multiplied by the number of operations of the recipe that can be altered in a single trial.

It is assumed that the recipe adoption process is very simple. The firm is myopic: if

20   ω_{t-1} is the prevailing recipe and ω'_{t-1} is the trial recipe, then the prevailing technology for period t, ω_t, will be given by

$$\omega_t = \begin{cases} \omega_{t-1} & \text{if } \ell(\omega_{t-1}) \le \ell(\omega'_{t-1}) \\ \omega'_{t-1} & \text{if } \ell(\omega_{t-1}) > \ell(\omega'_{t-1}) \end{cases}$$

25   and   (1)

$$\text{Prob}\left\{ \omega'_{t-1} = \omega^* \right\} = \begin{cases} 1/\#\tilde{N}_\delta(\omega_{t-1}) & \text{for } \omega^* \in \tilde{N}_\delta(\omega_{t-1}) \\ 0 & \text{for } \omega^* \notin \tilde{N}_\delta(\omega_{t-1}) \end{cases}$$

The system (1) defines the cost-reduction dynamics (see **FIG. 5**). This is a process in

30   which the firm moves from vertex to vertex of the technological graph Γ that underlies the IRF. If $\delta = 1$, then the firm moves along the edges of the graph Γ to the next vertex.

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For the nascent sub-recipes, it is assumed that the random variables $\phi^i(\omega^{i*})$ are independent and identically distributed (i.i.d.) and uniform on [0,1/n]. Hence if all the sub-recipes of ω are nascent, then the support of the random variable $\phi(\omega)$ is contained in [0, 1]. The density function of $\phi(\omega)$ is then ∩-shaped and symmetric about ½, the modal

5 labor requirement. The density is 0 for the extreme labor requirements, 0 and 1; otherwise it is positive. If ω^{i*} is currently available, then $\phi^i(\omega^{i*})$ is a degenerate random variable, in which all of the probability is massed on a scalar $\ell_i(\omega) \in [0, 1/n]$. If the recipe ω is currently available, then $\phi(\omega)$ is a degenerate random variable, in which all of the probability is massed on a scalar $\ell(\omega) \in [0,1]$. If m of the sub-recipes of the recipe ω are nascent (while the

10 remaining $n - m$ sub-recipes are currently available), then the support of $\phi(\omega)$ is $[\ell, \ell + (m/n)]$, where ℓ is a scalar in $[0, (n - m)/n]$. Probability $(n - m)/n$ is massed on ℓ. On the interval $(\ell, \ell + (m/n))$, the *probability density function* (p.d.f.) is ∩-shaped and symmetric if $m > 1$. (If $m = 2$, then the p.d.f. on $(\ell, \ell + (m/n))$ is a "tent.")

The assumption that the support of $\phi(\omega)$ is contained in [0,1] is not innocuous. The

15 fact that a zero labor requirement is possible could in principle be disturbing, but it is assumed that the probability of such a free lunch is zero. The assumed boundedness of unit cost is more serious. In actuality, many untried recipes turn out to be totally useless – i.e., with the realization $\ell(\omega) = +\infty$. The assumption of boundedness from above of unit labor requirements is an optimistic assumption about prior beliefs on untried recipes.

20 A formal description of experience curves and a review of the (largely empirical) literature on them is now given. The statistical outputs of the existing studies were used as inputs for the simulations of the disclosed embodiment. The existing experience curve analyses are not redone; instead the calculated regression coefficients and their standard errors are used as *observations*.

25 Few empirical phenomena in production economics or management science are as well documented as the *experience curve*. It depicts the decline in the firm's per-unit labor requirements (or costs) with the cumulative production of a given manufactured good. Following Wright's (1936) (T. P. Wright, Factors affecting the cost of airplanes, *Journal of the Aeronautical Sciences*, 2:122-128, 1936) study of the airframe industry, this pattern has been

30 investigated for many different goods and industries.

The usual parametric form of the experience curve is the power curve,

20

$$\ell_t = a Y_{t-1}^{-b} \qquad\qquad (2)$$

where $\ell_t = \ell(\omega_t)$ is the unit labor requirement for production run t, $Y_{t-1} = \sum_{s=1}^{t-1} y_s$, is the

cumulative output up to (but not including) run t, $b > 0$ is the learning coefficient, and

5    $a > 0$ is the labor needed to produce the first batch of the good. In the case of airframes, one
airframe is equal to one measured batch. In this case, Y_t would be the serial number of the
last airframe in production run t. The learning coefficient represents the rate at which the
productivity increases as the firm acquires "experience."

There is an impressive amount of empirical work on the engineering experience
10   curve. Wright's law is that unit cost ℓ_t is related to Y_{t-1}, cumulated output, by the power
function $\ell_t = a(Y_{t-1})^{-\frac{1}{3}}$, so that the progress ratio p is given by $p = 2^{-\frac{1}{3}} = 0.79$. If one allows
for a more general power law, one gets that

$$\ell_t = a(Y_{t-1})^{-b},$$

15   where $b > 0$ and $p = 2^{-b}$. The progress ratio p is a decreasing function of the exponent b, the
learning coefficient. Post-Wright empirical studies suggest that observed progress ratios live
in the range of about 60-95% with a mode of about 81-82%. The existing empirical literature
reports not only mean progress ratios but also their standard deviations.

A commonly used measure of productivity improvement is the *progress ratio*. The
20   progress ratio p is related to the learning coefficient b by

$$p = 2^{-b} \text{ or } b = -\log_2(p).$$

The percentage cost reduction when cumulative output is doubled is $100(1 - p)$. If $b = \frac{1}{3}$,
then $p = 0.79$ ("Wright's Law"), so the percentage cost reduction is 21%. If $b = 0.30$, then $p = 0.81$ (see the mode in **FIG. 17**). A small progress ratio is an indicator of rapid cost
25   improvement, while a higher progress ratio indicates less cost improvement. For example, a
p value of 0.85 (an "85% curve") means that per-unit cost falls by 15 percent when
cumulative output is doubled, $p = 0.60$ means per-unit cost falls by 40% when cumulative
output is doubled, $p = 1$ means that unit cost is constant, and $p > 1$ means that unit cost is
increasing.

30   The power law given in Equation (2) yields the straight line in log/log space

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$$\log \ell_i = \log a - b \log Y_{i-1},$$

where ℓ_i is the average unit labor requirement (or cost) of producing the goods with serial numbers greater than Y_{i-2} and less than or equal to Y_{i-1}.

5   In Table 1 (FIG. 6) are displayed some estimates of the learning coefficients and the progress ratios from firm-level experience curves (by industry). From this and the review articles by Conley (1970) (P. Conley, Experience curves as a planning tool, *IEEE Spectrum*, 7:63-68, 1970), Yelle (1979) (L.E. Yelle, The learning curve: Historical review and comprehensive survey, *Decision Sciences*, 10, January 1979), Dutton and Thomas (1984)

10   (J.M. Dutton and A. Thomas, Treating progress functions as a managerial opportunity, *Academy of Management Review*, 9(2):235-247, 1984), Muth (1986) (J. Muth, Search theory and the manufacturing progress function, *Management Science*, 32:948-962, 1986), and Argote and Epple (1990) (L. Argote and D. Epple, Learning curves in manufacturing, *Science*, 247:920-924, 1990), one concludes that the salient characteristics of experience

15   curves are: (1) The distribution of progress ratios ranges from 55% (rapid progress) to 105% (slow— indeed negative!—progress) and "centers" on about 81-82%. See Table 1 (FIG. 6) and FIG. 17. (2) Distinct production processes and goods are associated with their own ranges of values for the estimated progress ratio p. See Table 1 (FIG. 6). (3) There is variation in progress ratios not only among firms in the same industry, but also among

20   different plants operated by a single firm. See Alchian (1963) (A. Alchian, Reliability of progress curves in airframe production. *Econometrica*, 31:679-693, 1963), Dutton and Thomas (1984, pp.236-239), and Epple, Argot and Devadas (1991) (D. Epple, L. Argote, and R. Devadas, Organizational learning curves: A method for investigating intra-plant transfer of knowledge acquired through learning-by-doing, *Organization Science*, 2(1): 58-70,

25   February 1991). (4) The specification of the OLS (ordinary least-squares) statistical model, $\ell_i = aY^{-b}_{i-1}\varepsilon_i$ with $\log \varepsilon_i - N(0, \sigma)$, is imperfect in (at least) two ways: (A)There are "*plateau effects*" in the observed data (see Abernathy and Wayne (1974) (W.J. Abernathy and K. Wayne, Limits of the learning curve, *Harvard Business Review*, September-October 1974)): (i) Improvements occur after relatively long stretches of constant labor requirements, and (ii)

30   improvements in labor productivity cease beyond some (sufficiently large) cumulative output (see Conway and Schultz (1959) (R.W. Conway and A. Schultz Jr., The manufacturing

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progress function, *Journal of Industrial Engineering*, 10:39-54, 1959) and Baloff (1971) (N. Baloff, Extension of the learning curve – some empirical results, *Operational Research Quarterly*, 22:329-340, 1971). The (hypothetical) empirical experience curve of **FIG. 18** illustrates both types of plateauing. The labor requirement ℓ is falling from $t = 0$ to $t = 10$, but

5 not strictly monotonically. There is an interior plateau at batches 3-5. Productivity improvements cease after batch 7, providing a terminal plateau. (B) There is *curvature mis-specification* (see Levhari and Sheshinski (1973) (D. Levhari and E. Sheshinski, Experience and productivity in the Israeli diamond industry, *Econometrica*, 41:239-253, 1973) and Epple et al. (1991, pp. 65-69)). Instead of a straight line in log/log space, the data suggest that an S-

10 shaped curve would often (but not always) fit better: often the data suggest concavity of the function over the early batches, but convexity of the function over the later batches. This could be called the *SFS* phenomenon. Cost improvement is first *slower*, then *faster*, and finally *slower* than suggested by the straight-line log/log fit. See **FIG. 19**. (In many observed production runs and many experiments, the right-hand tail of the productivity plot is truncated

15 before the suggested convex range of the function can be observed.) (5) Industry experience curves (in which the data on cost as a function of cumulative output is averaged over several firms) are smoother than the corresponding single-firm experience curves, which in turn are smoother than single-plant experience curves. There are fewer plateaus and the lengths of the interior plateaus are shorter for the averaged data. See the survey by Dutton and Thomas

20 (1984).

An analysis of the comparative dynamics of the disclosed model of shop floor learning-by-doing is given next. In particular, the effects of varying the following basic parameters of the model are described: the number of operations n; the number of instructions per operation s; the externality parameter e; the maximum number of steps per

25 trial δ; the number of trials per batch τ; and the length of the production run T, on the two basic predictions of the model: the sample mean $\bar{p}$ of the estimated progress ratios and the sample standard deviation s_p of the estimated progress ratios, and on two measures of model mis-specification: plateauing (or its inverse, the improvement percentage z); and curvature (or SFS) in log/log space.

30 The basic parameters of the model are summarized in Table 2 (**FIG. 7**). The third column gives the range over which each parameter is defined. Non-integer values of τ would

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have been possible. The interpretation of (say) $\tau = \frac{1}{3}$ is that a trial occurs once in every three production runs.

Computations were performed on a Dell Dimension XPS Pro 200 PC with a Pentium Pro 200 MHz processor and Windows 95 operating system (version 4.00.950 B). The core
5 program used was written by Bennett Levitan, building on work by William Macready and Terry Jones. Regressions were performed, summary statistics were computed, and plots were generated with SPSS Windows version 7.5 and Matlab Windows version 4.0. The routines used to compute and generate the simulations from the random parameter set were written by Auerswald. Both Levitan's and Auerswald's programs incorporate a random number
10 generator written by Terry Jones, based on the algorithm of Knuth (1981, pp. 171-72).

The most fundamental unit of analysis is a single realization of the production run, examples of which are displayed in FIGS. 20-23. A "point" in one of these figures gives the normalized log of the unit labor requirement for the currently prevailing technology. The horizontal axis is the log of time (or, equivalently, the log of the accumulated number of
15 quality-control batches to date). A production run is then a "walk" on a landscape (i.e., a realization $\ell(\omega)$ of the IRF $\phi(\omega)$). The line in one of these figures is the OLS fit to the points in the figure.

There are many approaches to defining in detail the e "externality" connections from one operation in a given recipe to other operations in that recipe. If one had some
20 engineering information about these connections one could use this prior information. In the absence of engineering priors, one may draw for *each production run* t the $(e-1)$ connections to operation i uniformly (without replacement) from the set of all operations other than i.

Fixing these e connections, a preferred embodiment first computes and stores the
25 realizations of the s^e random variables $\phi^i(\omega^{i1},..., \omega^{ie})$, (where operation i is fixed but the sub-recipe $(\omega^{i1},..., \omega^{ie})$ varies). It does this for each of the operations $i = 1,..., n$. The number of vectors needed to compute the landscape $\ell(\omega)$ is thus reduced from s^n to ns^e, (In the case of $n = 10$, $s = 3$, $e = 3$, one gets that $s^n = 3^{10} > 270 = (10)3^3 = ns^e$.)

The landscape $\ell(\omega)$ and the method of "walking" on this landscape have now been
30 completely defined. All that remains to be specified is the starting point (on the landscape) for the walk. It should be clear to those of ordinary skill in the art that for some applications,

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the starting point might be given by information about the production experience of competitors or suggestions from the firm's R&D department. In the absence of such prior information, one can merely pick one recipe randomly (with uniform probabilities over the s^n recipes in the set Ω) to be the starting point.

5   In a preferred embodiment, the log (unless otherwise indicated, log denotes the natural logarithm) of the labor requirement is re-normalized so that the adjusted log of the initial unit labor requirement is 1.0. The relationship between the labor requirement ℓ and the adjusted log labor requirement is given by:

$$adjusted\ log\ labor\ requirement = 1 + \log \ell.$$

10 The adjusted log labor requirement is negative for $\ell < 0.36787$. Negative values of this convenient measure cause no problems (though it would be economic nonsense if *unadjusted* ℓ were to be negative).

For each realization of a single production run, the estimated progress ratio $\hat{p}$ is given by

$$\hat{p} = 2^{-\hat{b}}$$

15 where $\hat{b}$ is the OLS estimate of the learning parameter b, i.e., $\hat{b}$ is the absolute value of the slope of the regression line. If $\tau = 1$, then the labor requirement at t, ℓ_t, and cumulative output at t, Y_t, correspond to what has been used for estimating $\hat{b}$ and $\hat{p}$ in the known art on experience curves. If $\tau \neq 1$, then Y_t will *not* be equal to the cumulative number of trials that define time in the model. The relationship between t (cumulative number of trials), Y_t (cumulative output), and τ (number of trials per measured batch) is

$$\tau = \frac{t}{Y_t}$$

In general Y_t is the appropriate measure of "economic time" for a given production run. If

25 $\tau=1$, "calendar time" t and "economic time" Y_t are the same. Otherwise, t must be adjusted to measure economic time. In computational terms, if t is fixed, then increasing τ decreases the number of simulated points. Trials take place and labor requirements are modified at each date t, but not all modifications are recorded. For example, with $\tau = 20$ (and $B = 1$ as assumed), calculation of the per-unit labor requirement would not occur until after the 20th

30 unit was produced, then again not until the 40th unit, and so on. Under the assumption of $t =$

25

1000 and $\tau = 1$, 1,000 data points in a given simulated experience curve were observed, but with $\tau = 20$ only 50 data points would be observed.

Another quantity that can be used in measuring cumulative increases in productivity is ℓ_T, the labor requirement after τ trials (or the "terminal" labor requirement, for short). The

5 focus is on the path of productivity increases, not on the initial labor requirement, so one normalizes $\ell_0 = 1$. The terminal labor requirement ℓ_T is usually (but not always!) inversely related to b : usually, the lower the final labor requirement the steeper is the experience curve. If there were no specification error of the experience curve, this would always be the case and ℓ_T would be an uninteresting statistic. Consider, however, the case in which there

10 are huge productivity increases in the first few periods after which the labor requirements asymptote. For T large, b would then be small because OLS would heavily weight the asymptote in this case. Here the fact that ℓ_T is small indicates that the estimated learning coefficient b (and hence the estimated progress ratio p) might be misleading.

In order to capture the extent of plateauing, one computes an (inverse) statistic z,

15 *improvements percentage per measured batch*, defined by

$$z = \frac{\tau \times \text{number of observed improvements}}{T}$$

The measure z is preferable to direct measures of plateauing (e.g., average plateau length)

20 because it is less sensitive to the distortions caused by the presence of a long final plateau. For experience curve analysis, the transient is of greater interest than the steady state, since in most real world cases, the rate of product replacement (due to, say, a new, superior product) is rapid relative to the exhaustion of productivity improvements for the original product. The total number of improvements observed by τ/T is weighted so that the measure will reflect,

25 not the absolute number of improvements found, but rather the likelihood that a new observation will be a productivity improvement.

Finally, in order to measure the extent of curvature mis-specification in the experience curve data, one estimates a second quadratic specification of the learning model:

$$\log \ell_Y = a_2 + b_2 \log Y + c_2 \log Y^2 + \varepsilon_t$$

30 where ℓ_Y is the labor requirement after cumulative production of Y units. The magnitude, sign and level of significance for $\hat{c}_2$, the estimate of c_2, give an indication of the extent of curvature mis-specification in the standard log-log model. A negative and significant $\hat{c}_2$

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would suggest that the log-log form overstates the rate of early, relative to later, productivity improvements. This is not a test of the full SFS effect. A negative value of c_2 suggests the observed run exhibits SF (first *slow* and then *fast*). Often the second S of SFS is outside the observed or calculated production run.

5   FIG. 20 represents a single learning curve for the "base" parameter vector: $n = 100$, $s = 10$, $e = 5$, $\tau = 1$, $\delta = 1$, $T = 1000$. These parameters were chosen to be reasonable for the described simulation. The priors about the validity of these parameters were not very strong. Hence sampling was done widely in the space of parameters, but in many of the experiments (173 experiments out of 423) the parameter space was sampled by moving only one

10   parameter at a time while holding the others at one of its base values. This is a particular type of sensitivity analysis.

In FIG. 20, the landscape is not perfectly smooth since $e = 5 > 1$. Plateauing is also evident throughout the production run. This is confirmed by the small value of z: only 6.2% of the trials lead to improvements in productivity. Overall productivity increase is moderate;

15   this is confirmed by the estimated progress ratio of 87.5%. There is also a positive SFS (curvature) effect, but it is not strong. In FIG. 21, two changes have been made in the parameters to set $e = 1$ and $n = 1,000$, so that the landscape is now as smooth as possible (because $e = 1$) and a single step improvement is likely to be small (because n is large *and $e =*

1). With the smooth landscape, plateauing (except for terminal plateauing) is so reduced that

20   it cannot be detected by eyeballing the figure. This is confirmed by the value of z: 42.3% of the trials result in productivity improvements. The estimated progress ratio of 94.6% indicates a relatively slow rate of productivity improvement, but $\hat{p}$ is upward biased because of the SFS effect, which seems to be more pronounced. (One must be careful, however, not to visually overweight the sparse plot in log units for the early periods (relative to the later

25   periods) in evaluating the SFS effect; the later data points are more crowded together than the early data points.) If the quadratic model were fitted to the plot in FIG. 21, one would expect c_2 to be significantly negative, since the plot suggests a concave function. (Note that the plot displays clearly the SF of SFS, but that the final S is not displayed in the plot, because of the truncation at log batch number $= 7$.)

30   In FIG. 22, the externality parameter e has been increased to its maximum relative to n ($e = n = 100$). The landscape is very rugged because of the very large e. Consequently,

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plateauing is very strong (only 1% of the trials lead to improvements) and overall progress is very small ($\hat{p} = 97.8$).

In **FIG. 23**, $e = 1$ so there are no externalities to cause a rough landscape. But n is reduced to 10 while s is increased to 100. The number of operations is few, so that a change

5   in any operation can be expected to have a large impact on the labor requirement. This is reflected by very rapid overall productivity improvement, which is confirmed by the value of the estimated progress ratio of 60.2%. Plateauing is evident (z is only 4.4%). The SFS effect is evident, although the caution against over-weighting the sparse early realizations also applies here.

10   In **FIGS. 20-23**, only one production run was used. Each of **FIGS. 24-28** presents averaged data of multiple production runs based on a given vector of parameters. **FIG. 24** is nearly the same as **FIG. 20** except that in **FIG. 24** the data is averaged over 20 separate production runs. The vertical axis in **FIG. 24** measures the adjusted log labor requirement of the industry average over 20 firms or, alternatively the adjusted log labor requirement of the

15   firm average over 20 plants. The most important difference between the outputs in **FIG. 24** and **FIG. 20** is the degree of plateauing. The curve in **FIG. 24** is far smoother than the curve in **FIG. 20**. Plateauing is evident in **FIG. 20**, while it is barely discernible in **FIG. 24**. This is confirmed by the z statistics: in the single run case only 6.2% of the trials resulted in improvement, while in the 20-run average, 65% of the trials resulted in improvement. The

20   estimated progress ratio (87.5) for the single firm walk is very close to the estimated progress ratio (86.5) for the average walk of the 20 firms. The small, but positive curvature effect seems to be the same for **FIGS.** 20 and 24.

FIG. 21 is based on a correlated landscape ($e = 1$) and hence the single run is quite smooth, but some plateauing is discernible. The average walk shown in **FIG. 25** is even

25   smoother. For the single run depicted in **FIG. 21**, p = 94.6% and $z = 42.3\%$. For the averaged run depicted in **FIG. 25**, $p = 94.6\%$ and $z = 99.9\%$. The strong positive curvature effects in the two figures are nearly identical.

The landscape behind **FIG. 22** is very rugged ($e = 100$). Plateauing is dominant. Only 1% of the trials result in improvements. **FIG. 26** is the 20-firm average run using **FIG.**

30   22 parameters. Plateauing is reduced; 7.8% of the trials result in improvement. **FIG. 27** is the 50-firm average run based on the same rugged landscape data. Plateauing is further

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reduced; 15.7% of the trials result in improvements. The estimated progress ratio for each of the 3 cases (FIGS. 22, 26, 27) is about 98%. It is difficult to judge the SFS effect when there is so much plateauing, but it is positive and one could argue that the effect is constant across FIGS. 22, 26, and 27.

5   FIG. 28 is the 20-firm average related to the single firm walk in FIG. 23. The averaging reduces plateauing (increases z), has no effect on the progress ratio or the terminal labor requirement, and seems to reduce the SFS effect but only slightly.

From these experiments, the following conclusions may be reached: (a) averaging profoundly reduces plateauing (and increases z); (b) averaging does not substantially affect 10 the estimated progress ratio; and (c) averaging does not seem to have a strong effect on curvature or SFS. These are consistent with actual observations of the firm and industry experience.

Many of the qualitative features of experience curves—ranges of values of p, presence of plateaus, different learning rates for the same or similar goods—can be discerned by exam-15 ining single realizations of learning curves. However, to study the full effects of changes in the values of the underlying parameters on the predictions of the model, one needs to compute more than one realization per set of parameter values. Consequently, for each chosen parameter vector in the experiment, 20 independent realizations were computed.

In the simulations, computing 20 different realizations means running the simulation 20 program using the same parameter set, but with 20 different random seeds. A new random seed yields a new realization of the externality connection among the operations, a new realization of the landscape $\ell(\omega)$, a new starting point on the landscape and hence a new experience curve. Hence in this class of models the "maximum degree of randomness" between different realizations of the experience curve was chosen. (Even if the sets of 25 connections E_i ($i = 1,...,n$), the realization of the landscape, $\ell(\omega)$, and the starting recipe, ω_0, were all to be held constant, different realizations of the experience curve would still be possible (indeed almost certain) due to different sequences of recipe sampling.) The sample standard deviations computed are therefore likely to be biased upward.

The set of potentially interesting parameters is large. Computations were restricted to 30 the following grid-like parameter space:

$$n = 1, 10, 20, 50, 100, 500, 1000;$$

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- $s = 2, 10, 25, 50, 75, 100;$
- $e = 1, 2, 3, 5, 6, 7, 8, 9, 10, 25, 50, 75, 100$ (but with the constraint that $e \leq n$);
- $\delta = 1, 2, 4, 10, 25, 50, 75, 100$ (but with the constraint that $\delta \leq n$);
- $\tau = 1, 10, 50, 100, 250$ (but with the constraint that $\tau \leq T$);

5
- $T = 100, 500, 1000, 5000.$

Two subsets of parameters were used. The first subset, the focused set of parameter values, reflects the priors informed by a review of the empirical literature, introspection about production processes, and comparisons with the modeling in evolutionary biology. To achieve the rapid productivity increases that have been observed, rather small values of e and

10 δ relative to n were focused on (in particular: $e = 1$ and 5 and $\delta = 1$), as well as relatively low values of s (in particular: $s = 2$ and 10). Actually 5 is not a small value for the parameter e when $n = 100$ or $n = 1,000$. In fact, $e — 1$ gives the number of externalities per operation, so the number of externalities per recipe would be $n(e — 1)$. The length of the run was frequently set at $T = 1000$. This was chosen to reduce or eliminate the effects of the terminal

15 plateau. See the center panel in **FIG. 35**, which confirms our strategy. For $s = 10, n = 100, e = 5$, the mean progress ratio is smallest for $T = 1000$. For $T > 1000$ the effect of the terminal plateau is to reduce the progress ratio. For many runs, $n = 100$ was adopted for the number of operations. For many runs $\tau = 1$ was adopted. The base cases for the focused runs are then:

- $(n = 100, s = 2, e = 1, \delta = 1, \tau = 1, T = 1000)$

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- $(n = 100, s = 10, e = 1, \delta = 1, \tau = 1, T = 1000)$
- $(n = 100, s = 2, e = 5, \delta = 1, \tau = 1, T = 1000)$
- $(n = 100, s = 10, e = 5, \delta = 1, \tau = 1, T = 1000)$

The focused parameter set was constructed from the above four base case vectors by varying the six parameters one at a time. Summary statistics for the focused parameter set are

25 given in Table 3 (**FIG. 8**). There are 173 parameter vectors in the focused set. For each vector there are 20 runs, so the total number of runs is 3,460.

If the parameter space were only of dimension 2 and there were only a single base case, the method of choosing parameters would be that described in **FIG. 29**. Say the base parameter vector is given as row 3, column 5 of the simple 10 x 10 matrix recipe set in **FIG.**

30 **29**. Then the set of focused parameters is the union of the set of recipes having a row-3 component with the set of recipes having a column-5 component. The focused parameter set

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is in the shaded "cross." The advantage of the focused parameter method is that one begins with a set of reasonable base parameters and then tests the sensitivity of the predictions to each of these parameters varied one at a time from each of the base vectors. The parameters are often set at "extreme" values to test parameter sensitivity. On the other hand, as can be

5   seen from FIG. 29, this selection of parameter points is clearly "statistically inefficient" and results could be sensitive to a choice of base parameter vectors. As a counter to these potential biases, experience curves based on a *random set of parameter values* were also simulated. The random parameter vectors were chosen as follows: *n* drawn uniformly from {1,...,1000}; *s* drawn uniformly from {1,...,100}; *e* drawn uniformly from {1,...,

10 min(10,n/2)}; δ drawn uniformly from {1,...,min(10,n/2)}; *r* drawn uniformly from {1,...,10}; and $T = 1000$.

The number of parameter vectors in the set of random parameters is 250. There are 20 runs per vector. Hence there are 5000 runs in all for the random parameter set. Summary statistics for the random parameter set are given in Table 4 (**FIG. 9**).

15 The parameters selected for the focused set suffer from "**FIG. 29** bias"; the parameters selected for the random set do not. These are not the only differences between the two parameter sets. See Tables 3 and 4. For the parameters *n* and *s*, the ranges and the means are greater in the random set than in the focused set, but this is reversed for *e* and δ. In the focused set, τ was varied between 10 and 1000, but in the "random" set T was fixed at

20 1000. The means of the predicted $\bar{p}$s are similar, but the range and the standard deviations of the $\bar{p}$s are much larger for the focused set. The mean of the $\bar{z}$s, the range, and the standard deviation are larger for the focused set. Curvature predictions and labor requirements were not assembled for the focused set.

The effects of varying the parameters of the model were evaluated using two scoring

25 methods. For the focused parameter set, judgment from studying all the relevant simulations was used. This is called here "the *eyeball method* of scoring" parameter effects. The eyeball method is very illuminating, but it does require judgment from the modeler. As a guard against human bias, simple OLS *regression scoring* for the focused parameters (173 experiments) and the random parameters (250 experiments) was also used. The basic

30 disadvantage to OLS scoring is that the results are "global": "extreme" parameter values are

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probably overweighted. Furthermore, subtle non-monotonic interrelationships are obscured by the simple functional forms used for OLS scoring.

The effect of varying a particular parameter on one of the predictions (or "results") of the model typically depends on the interplay of two effects: (1) the effect of the parameter 5 change on the size of the recipe space and (2) the effect of the change on the trial (or, recipe sampling) mechanics. Typically, these effects are too complicated to permit an analytic solution, especially since the bias is on the short term and the medium term. Computation is called for.

The first effect of increasing n is to increase the number of recipes, s^n. This effect 10 (especially for large T) should tend to increase *long-term* productivity improvement. On the other hand, increasing n decreases (especially when e is small) the expected cost reduction of one-step or few-step changes in the recipe on unit labor costs, because—given the assumption of additive unit costs—with more operations each operation contributes less to the overall cost. This suggests that increasing n might decrease the rate of short-term (and medium- 15 term) productivity improvement.

In FIG. 30, the sample means, $\bar{p}$, (over 20 runs) and the sample standard deviations, are plotted. It was observed that for small n the effect of increasing n is to decrease $\bar{p}$ (i.e., to increase the mean rate of productivity improvement), but for larger n, the effect on $\bar{p}$ of increasing n is positive. As n becomes even larger, the effect on $\bar{p}$ of increasing n attenuates. 20 The standard deviation, s_p, is decreasing in n.

The sole effect of increasing the parameter s on the rate of productivity improvement is through increasing the size of the recipe space and hence through increasing long-run productivity improvement. From FIG. 31, it is apparent that this is confirmed. Increasing s substantially decreases $\bar{p}$ while slightly increasing s_p, but each effect eventually attenuates as s 25 becomes very large. In particular, for s larger than 50 the effects on $\bar{p}$ of increasing s are negligible.

The most obvious effect of increasing e is increasing the ruggedness of the landscape, thereby reducing the effectiveness of the myopic recipe sampling procedure. As e increases the number of local optima in the landscape increases, thus the probability of being "trapped" 30 increases. This reasoning suggests that $\bar{p}$ would tend to be increasing in e. On the other hand, increasing the parameter e increases the number of cost-relevant *sub-recipes* (equal to

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ns^e), and thus reduces the expected value of the global minimum labor requirement. Furthermore, increasing e has the effect of speeding the rate of experimentation, since each trial modifies the contribution to the labor requirement of not one, but e different operations within the production recipe. For these last two (related) reasons, one might expect $\bar{p}$ to tend

5   to be decreasing in e.

Results for e are given in **FIG. 32** and **FIG. 33**. If the recipe space is relatively small ($s < 6$, $n = 100$), then increasing e seems to decrease $\bar{p}$ for e small, although the standard errors suggest that one should be cautious in making this conclusion. The smallest $\bar{p}$ (and the lowest terminal labor requirements) seems to occur at values of $e \approx 5$. If the recipe space is

10 larger ($s > 6$, $n = 100$), then $\bar{p}$ and ℓ_T are clearly monotonically increasing in e. For s large, the transitory effects of e depend on how much progress has already been made. If $\ell(\omega_t)$ is above the expected value of $\phi(\omega)$ over all of Ω, then increasing e (for small values of e) increases the expected rate of short-run productivity improvement. On the other hand, if $\ell(\omega_t)$ is below the expected value of $\phi(\omega)$, then increasing e (for small values of e), decreases the expected

15 rate of productivity improvement. See Auerswald (1998, Chapter 3) (P. Auerswald, *Organizational Learning, Production Intranalities and Industry Evolution*, University of Washington Ph.D. thesis, 1998). This is because the ruggedness is helpful in "bad" (high ℓ) neighborhoods, but hurtful in good (low ℓ) neighborhoods. This phenomenon is seen in the simulations. This argument also suggests that increasing e tends to increase the curvature

20 mis-specification. This effect is present in the simulations.

Taking bigger steps on a given landscape is somewhat like walking with smaller steps on a more rugged landscape. Hence, increasing δ should be analogous to increasing e. Like e, increasing δ increases $\bar{p}$ except for some cases with δ small; see **FIG. 34**. There are suggestions in the data that for small δ and appropriately chosen values of the other

25 parameters, $\bar{p}$ is decreasing in δ. Indeed, the parameters e and δ are close cousins. (They would be even closer if by increasing δ one would be increasing the size of *every* "step" rather than merely increasing the maximum step size.)

The length of the production run would not affect $\bar{p}$ if the model were perfectly specified, in particular, if a power law fitted the data well. Varying T provides a method for

30 analyzing the curvature mis-specification of the experience curve. See **FIG. 35**. For the case with a relatively small recipe space ($s = 10$, $n = 100$) and a relatively rugged landscape ($e =$

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5), the SFS effect is pronounced. The progress ratio falls from $T = 1$ to about $T = 1000$ and then gradually rises out to $T = 5000$. This pattern is also suggested for the case: $s = 100$, $n = 10$, $e = 5$, but the standard errors are too large for confidence. For the case: $s = 10$, $n = 1000$, $e = 1$, the recipe space is very large and the landscape is smooth. The estimated mean

5   progress ratio is monotonically decreasing (with small standard errors) over $T = 1$ to $T = 5000$. The recipe space is so large in this case that the effects of the second *Slow* response do not kick in sufficiently to increase $\bar{p}$ even at $T = 5000$.

The statistic z (percent of trials that result in improved productivity) is an inverse measure of plateauing. Increasing the number of recipes by increasing n decreases plateauing

10   with no discernible effect on the standard deviation s_z (see FIG. 37). Increasing the number of recipes by increasing s decreases plateauing but increases (except for large values of s) the standard deviation s_z. Increasing e, in general, increases plateauing (See FIG. 38). Increasing δ, in general, increases plateauing, but for small e, s, and δ, it appears that increasing δ reduces plateauing (see FIG. 39).

15   The positive curvature effect is pronounced in FIGS. 21 & 25 (smooth landscape cases with: $n = 1000$, $s = 10$, $e = 1$, $\delta = 1$, $\tau = 1$, $T = 1000$). Linear OLS in log/log space at first overstates the actual rate of productivity improvement and then understates it. Ultimately (because of the terminal plateau), linear OLS in log/log space can be expected to overstate the rate of productivity improvement. This is not surprising for smooth, many-

20   recipe landscapes with n large and δ small. Because n is large, for each improvement the expected reduction in the labor requirement is small. Because the landscape is smooth, the probability of finding an improvement on any given trial is relatively high. The large recipe set implies that the stock of potential improvements is being exhausted at a very slow rate. Hence the resulting productivity plot is likely to be nearly linear in *natural units*. Therefore it

25   will be strongly concave to the origin in log/log space. This is in agreement with the results of the model and with observed experience curves. The data behind experience curves frequently show curvature bias, usually positive (indicating a concave function), but sometimes negative (indicating a convex function). In the discussion above on the effects of T on $\bar{p}$ and s_p, separate simulation evidence on curvature and SFS are also provided.

30   For the two parameter sets (focused and random), each of the predictions of the model was regressed on the parameters of the model. More efficient grids and pooling of data are

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known to those of ordinary skill in the art. See, e.g., Judd (1998, Chapter 9) (K. Judd, *Numerical Methods in Economics and Finance*, 1998) on acceleration techniques for Monte Carlo experiments. Various functional forms were employed. The results are not very sensitive to the functional form. Reported here are the regression results for cases in which

5 the prediction in natural units is regressed on log values of the parameters.

In Table 5 (FIG. 10) and Table 6 (FIG. 11) are summarized the results of OLS scoring for prediction of $\bar{p}$. The R^2s are not very high. This is probably because the functional form being used is highly mis-specified. Eyeball scoring indicated that the interaction effects of the parameters can be quite subtle and that the effects of varying even a

10 single parameter are not monotone. Nonetheless, the t tests yield high levels of significance for most of the parameters. The R^2s and the t tests are more favorable for the random set. This is probably because the random set does not suffer from "FIG. 29 bias," i.e., (1) the data is more dispersed for the random set and (2) the focused set conditions more on the interesting small values for which the results tend to be non-monotone in the parameters.

15 Both OLS scorings predict that increasing either of the two cousin parameters—e and δ—will increase $\bar{p}$. From the focused parameter set, the prediction is that increasing s decreases $\bar{p}$. The prediction from the random parameter set is the same but at a lower level of significance. From the random set, there is a weak prediction that increasing τ increases $\bar{p}$ from the focused set, the coefficient is not significant. This weak effect of τ on $\bar{p}$ is probably because of mis-

20 specification of the experience curve: as τ is increased initial productivity improvements can be so rapid that the terminal plateau is reached quickly. The estimated progress ratio then gives a downward biased estimate of the "actual rate of productivity improvement." Another reason that the τ effects might appear to be weaker than expected is that the model of reporting and implementation of quality control trials might not be the best one. For the

25 focused parameter set, increasing the parameter n decreases the prediction $\bar{p}$. For the random parameter set, increasing n strongly increases $\bar{p}$. This can be explained by the fact that the random parameter set contains bigger ns than does the focused parameter set; see Tables 3 and 4. Eyeball scoring strongly suggests that $\bar{p}$ is not monotone in n. For smaller n, $\bar{p}$ is decreasing in n. For larger n, $\bar{p}$ is increasing in n.

30 Tables 7-8 (FIGS. 12-13) report the prediction of the standard deviation s_p. Increasing n reduces the predicted s_p. Increasing s increases the s_p predicted from focused

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parameters. Increasing e decreases the predictions of s_p. The predicted effects of varying δ differ between the two parameter sets. Increasing τ decreases the prediction of s_p.

See Table 9 (FIG. 14). Increasing n increases the predicted mean improvement percentage, $\bar{z}$ while increasing e or δ decreases $\bar{z}$.

5   If $\bar{c}_2$ is negative then the estimated quadratic experience curve is a concave function in log/log space. See Table 10. Increasing n, s, or τ decreases predicted $\bar{c}_2$ and hence increases the curvature effect. Increasing e or δ increases the predicted $\bar{c}_2$ and hence decreases the curvature effect.

The observed values of $\bar{p}$ (from actual firms and industries) are likely to be biased

10   downward, since estimates of the progress ratio also pick up the effects of increasing returns to scale in production and of the development (the D of R&D) activity devoted to improvements in production efficiencies.

It is straightforward to test the predictions of a model in the following way: (1) take some learning curves for a particular industry; (2) adjust the reported statistics (such as $\bar{p}$ and

15   s_p) for the numbers of observations in each run; then (3) find in the set of predictions the parameter sets for which the predictions are near the observed statistics.

FIG. 40 discloses a representative computer system 4010 in conjunction with which the embodiments of the present invention may be implemented. Computer system 4010 may be a personal computer, workstation, or a larger system such as a minicomputer. However,

20   one skilled in the art of computer systems will understand that the present invention is not limited to a particular class or model of computer.

As shown in FIG. 40, representative computer system 4010 includes a central processing unit (CPU) 4012, a memory unit 4014, one or more storage devices 4016, an input device 4018, an output device 4020, and communication interface 4022. A system bus 4024

25   is provided for communications between these elements. Computer system 4010 may additionally function through use of an operating system such as Windows, DOS, or UNIX. However, one skilled in the art of computer systems will understand that the present invention is not limited to a particular configuration or operating system.

Storage devices 4016 may illustratively include one or more floppy or hard disk

30   drives, CD-ROMs, DVDs, or tapes. Input device 4018 comprises a keyboard, mouse, microphone, or other similar device. Output device 4020 is a computer monitor or any other

known computer output device. Communication interface 4022 may be a modem, a network

interface, or other connection to external electronic devices, such as a serial or parallel port

        While the subject invention has been particularly described with reference to a

preferred embodiment, it will be appreciated by those of ordinary skill in the art that various

5 changes and modifications may be made without departing from the spirit and scope of the

method. For example, the learning-by-doing model is sufficiently rich to match the reported

progress ratios from estimated experience curves. Those of ordinary skill in the art will

recognize that certain modifications are routine. For example, if one has data or priors on the

values of $\bar{p}, \bar{s}_p, \bar{z}, s_z, \bar{c}_2, s_{c'}$, or any of the other predictions for a particular plant, firm or

10 industry producing a specific good, then one can search for parameters n, s, e, δ and τ that

predict the data. One might also have priors on some of the parameters based on engineering

considerations or (especially in the case of T) market considerations. From this, one could

come up with a best explanation of the observed data. How well the modified model fit

observations and priors would measure the usefulness of the modified model of learning-by-

15 doing.

        It is intended that the appended claims be interpreted to cover the foregoing as well as

equivalents thereto.

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Claims

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CLAIMS

What is claimed is:

1.   A method for determining a production plan comprising the steps of:

        defining a plurality of production recipes such that each of said production recipes is a vector of n operations;

        selecting a current one of said production recipes;

        evaluating said current production recipe to determine its cost;

        modifying said current production recipe to create a trial production recipe;

        evaluating said trial production recipe to determine its cost; and

        assigning said trial production recipe to said current production recipe if said cost of said trial production recipe is less than said cost of said current production recipe.

2.   A method for determining a production plan as in claim 1 further comprising the steps of repeating said modifying said current production recipe step, said evaluating said trial production recipe step and said assigning said trial production recipe step.

3.   A method for determining a production plan as in claim 1 wherein each of said operations of said production recipes has s possible settings.

4.   A method for determining a production plan as in claim 1 wherein said cost of said production recipe is a labor cost.

5.   A method for determining a production plan as in claim 3 further comprising the step of defining a distance between said current production recipe and said trial production recipe.

6.   A method for determining a production plan as in claim 5 further comprising the step of defining a distance between a first one of said operations and a second one of said operations.

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7.   A method for determining a production plan as in claim 6 wherein said distance between said first one of said operations and said second one of said operations is defined as one if said first operation is not equal to said second operation and as zero otherwise.

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8.   A method for determining a production plan as in claim 6 wherein said distance between said first one of said operations and said second one of said operations is defined as the difference between said setting of said first operation and said setting of said second operation.

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9.   A method for determining a production plan as in claim 6 wherein said distance between said current production recipe and said trial production recipe is defined as a sum of said distance between said operations of said current production recipe and corresponding ones of said operations of said trial production recipe.

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10.   A method for determining a production plan as in claim 9 wherein said modifying step creates said trial production recipe in a neighborhood of said current production recipe.

20   11.   A method for determining a production plan as in claim 10 further comprising the step of defining said neighborhood of said current production recipe as those of said plurality of production recipes that are within a maximum distance from said current production recipe.

25   12.   A method for determining a production plan as in claim 1 further comprising the step of defining a connectivity indicator e_j^i as 1 if said setting of said operation i affects said cost of said operation j and 0 otherwise.

13.   A method for determining a production plan as in claim 12 further comprising
30 the step of defining a number e_i of operations that affect said cost of operation i as a sum of said connectivity indicators e_j^i over j from 1 to n.

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14.   A method for determining a production plan as in claim 13 further comprising the step of defining said number e_i as an externality parameter e.

15.   A method for determining a production plan as in claim 1 wherein said
5   modifying step is a non-committed experiment to said trial production recipe.

16.   A method for determining a production plan as in claim 1 wherein said modifying step is a commitment to said trial production.

10   17.   A method for determining a production plan as in claim 1 wherein said plurality of production recipes comprise backward technologies.

18.   A method for determining a production plan as in claim 1 wherein said plurality of production recipes comprise advanced technologies.
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19.   A method for determining a production plan as in claim 1 wherein said plurality of production recipes comprise nascent technologies.

20.   A method for predicting technological innovation comprising the steps of:
20   defining a model comprising:
                    a plurality of production recipes such that each of said production
                    recipes is a vector of n operations; and
                    a plurality of model parameters; and
            executing said model comprising the steps of:
25   selecting a current production recipe;
                    evaluating said current production recipe to determine its cost;
                    modifying said current production recipe to create a trail production
        recipe; and
                    assigning said trial production recipe to said current production recipe
30 if said cost of said trial production recipe is less than said cost of said current production
        recipe.

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21   A method for predicting technological innovation as in claim 20 wherein said executing said model step further comprises the steps of:

repeating said modifying said current production recipe step, said evaluating said trial production recipe step and said assigning said trail production recipe step.

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22.   A method for predicting technological innovation as in claim 20 wherein said plurality of model parameters comprise a possible settings parameter s defining a number of possible settings of said operations of said production recipes.

10   23.   A method for predicting technological innovation as in claim 20 wherein said plurality of model parameters comprise a connectivity indicator e_j^i defined as 1 if said setting of said operation i affects said cost of said operation j and 0 otherwise.

24.   A method for predicting technological innovation as in claim 23 wherein said 15 plurality of model parameters further comprise a number e_i of operations that affect said cost of operation i as a sum of said connectivity indicators e_j^i over j from 1 to n.

25.   A method for predicting technological innovation as in claim 24 wherein number e_i is defined to be a predetermined externality parameter e.

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26.   A method for predicting technological innovation as in claim 20 wherein said plurality of model parameters comprise a recipe distance between said current production recipe and said trial production recipe.

25   27.   A method for predicting technological innovation as in claim 26 wherein said plurality of model parameters further comprise an operation distance between a first one of said operations and a second one of said operations.

28.   A method for predicting technological innovation as in claim 27 wherein said 30 operation distance between said first one of said operations and said second one of said

41

operations is defined as one if said first operation is not equal to said second operation and as zero otherwise.

29.   A method for predicting technological innovation as in claim 27 wherein said operation distance between said first one of said operations and said second one of said operations is defined as the difference between said setting of said first operation and said setting of said second operation.

30.   A method for predicting technological innovation as in claim 27 wherein said recipe distance between said current production recipe and said trial production recipe is defined as a sum of said operation distances between said operations of said current production recipe and corresponding ones of said operations of said trial production recipe.

31.   A method for predicting technological innovation as in claim 26 wherein said plurality of model parameters further comprise a maximum distance from said current production recipe, said maximum distance defining a neighborhood of said current production recipe.

32.   A method for predicting technological innovation as in claim 31 wherein said modifying said current production recipe step comprises the step of creating a trial production recipe within said neighborhood of said current production recipe.

33.   A method for predicting technological innovation as in claim 20 wherein said plurality of model parameters further comprise a production run length.

34.   A method for predicting technological innovation as in claim 33 wherein said executing said model step is performed for said production run length.

35.   A method for predicting technological innovation as in claim 20 further comprising the step of defining values for said plurality of model parameters.

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36.   A method for predicting technological innovation as in claim 35 further comprising the step of defining at least one innovation prediction variable.

37.   A method for predicting technological innovation as in claim 36 wherein said
5 executing said model step determines values of said at least one innovation prediction variable corresponding to said values of said model parameters.

38.   A method for predicting technological innovation as in claim 36 wherein said at least one innovation prediction variable comprises a least one estimated progress ratio.
10

39.   A method for predicting technological innovation as in claim 38 wherein said at least one estimated progress ratio is defined as

$$\hat{p} = 2^{-\hat{b}},$$

wherein $\hat{b}$ is the OLS estimate of the learning coefficient b, wherein the learning coefficient b represents the rate at which productivity increases as a firm acquires experience, and wherein
15   (a) $\hat{b}$ is the absolute value of the slope of the regression line in log/log space given by

$$\log \ell_t = \log a - b \log Y_{t-1},$$

where $\ell_t = \ell(\omega_t)$ is a unit labor requirement for production run t, and wherein a is the labor
20 needed to produce a first batch of a good;

              (b) Y_{t-1} is defined by $Y_{t-1} = \sum_{s=1}^{t-1} y_s$, a cumulative output up to (but not

including) production run t;

              (c) $\tau = \dfrac{t}{Y_t}$, wherein a trials parameter τ satisfies
25

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$\tau = B/B$, wherein B is a measured batch size and B is a quality control batch size;

(d) $\ell(\omega_i)$ is defined to be the realization of a random field $\phi(\omega)$, wherein $\phi(\omega)$ is defined by

5

$$\phi(\omega) = \sum_{i=1}^{n} \phi^{i}(\omega),$$

wherein for each i, $\phi^{i}(\omega)$ is the unit labor cost of operation i.

10

20   40.   A method for predicting technological innovation as in claim 38 wherein said at least one innovation prediction variable further comprises a sample standard deviation of said at least one estimated progress ratio.

15   41.   A method for predicting technological innovation as in claim 35 further comprising the step of defining at least one measure of model misspecification.

42.   A method for predicting technological innovation as in claim 41 wherein said at least one measure of model misspecification comprises a plateau effect.

20

43.   A method for predicting technological innovation as in claim 42 wherein said at least one measure of model misspecification comprises a curvature misspecification.

44.   A method for predicting technological innovation as in claim 41 wherein said 25 executing said model step determines values of said at least one measure of model misspecification corresponding to said values of said model parameters.

45.   Computer executable software code stored on a computer readable medium, the code for determining a production plan, the code comprising:

30   code to define a plurality of production recipes such that each of said production recipes is a vector of n operations;

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code to select a current one of said production recipes;

code to evaluate said current production recipe to determine its cost;

code to modify said current production recipe to create a trial production

recipe;

5   code to evaluate said trial production recipe to determine its cost; and

code to assign said trial production recipe to said current production recipe if

said cost of said trial production recipe is less than said cost of said current production recipe.

46.   A programmed computer system for determining a production plan comprising

10 at least one memory having at least one region storing computer executable program code and

at least one processor for executing the program code stored in said memory, wherein the

program code includes:

code to define a plurality of production recipes such that each of said

production recipes is a vector of *n* operations;

15   code to select a current one of said production recipes;

code to evaluate said current production recipe to determine its cost;

code to modify said current production recipe to create a trial production

recipe;

code to evaluate said trial production recipe to determine its cost; and

20   code to assign said trial production recipe to said current production recipe if

said cost of said trial production recipe is less than said cost of said current production recipe.

47.   Computer executable software code stored on a computer readable medium,

the code for predicting technological innovation, the code comprising:

25   code to define a model comprising:

a plurality of production recipes such that each of said production

recipes is a vector of *n* operations; and

a plurality of model parameters; and

code to execute said model comprising:

30   code to select a current production recipe;

code to evaluate said current production recipe to determine its cost;

45

code to modify said current production recipe to create a trail

production recipe; and

code to assign said trial production recipe to said current production

recipe if said cost of said trial production recipe is less than said cost of said current

5   production recipe.

48.   A programmed computer system for predicting technological innovation

comprising at least one memory having at least one region storing computer executable

program code and at least one processor for executing the program code stored in said

10   memory, wherein the program code includes:

code to define a model comprising:

a plurality of production recipes such that each of said production

recipes is a vector of n operations; and

a plurality of model parameters; and

15   code to execute said model comprising:

code to select a current production recipe;

code to evaluate said current production recipe to determine its cost;

code to modify said current production recipe to create a trail

production recipe; and

20   code to assign said trial production recipe to said current production

recipe if said cost of said trial production recipe is less than said cost of said current

production recipe.

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FIG. 1

FIG. 2

$\omega=(1,3)$
$s=4$
$n=2$
$\delta=2$

$\square \longleftrightarrow N_2(\omega)$
$\circ \longleftrightarrow N_1(\omega)$
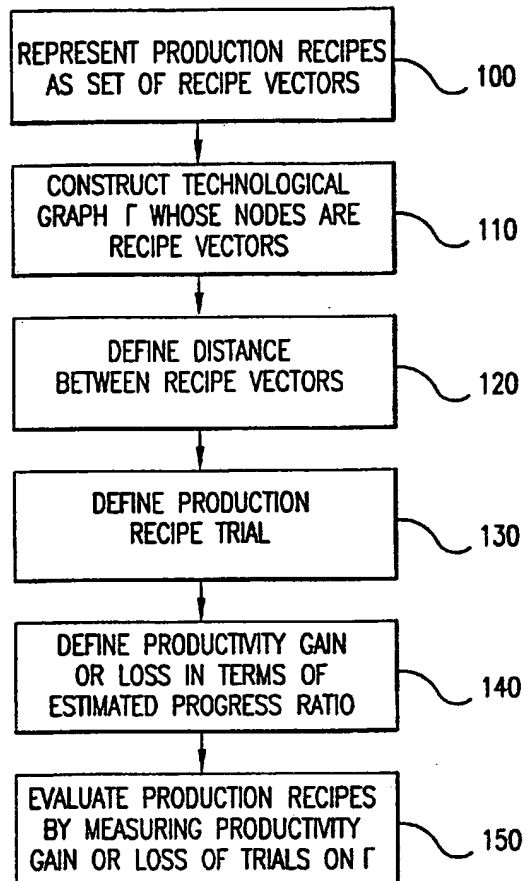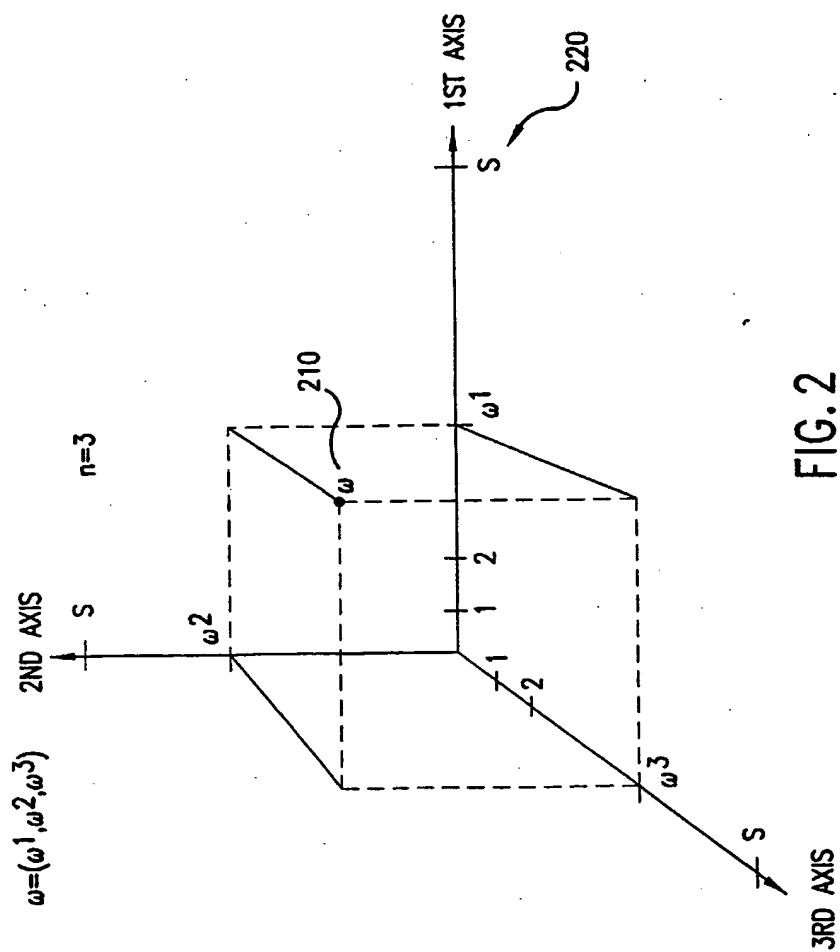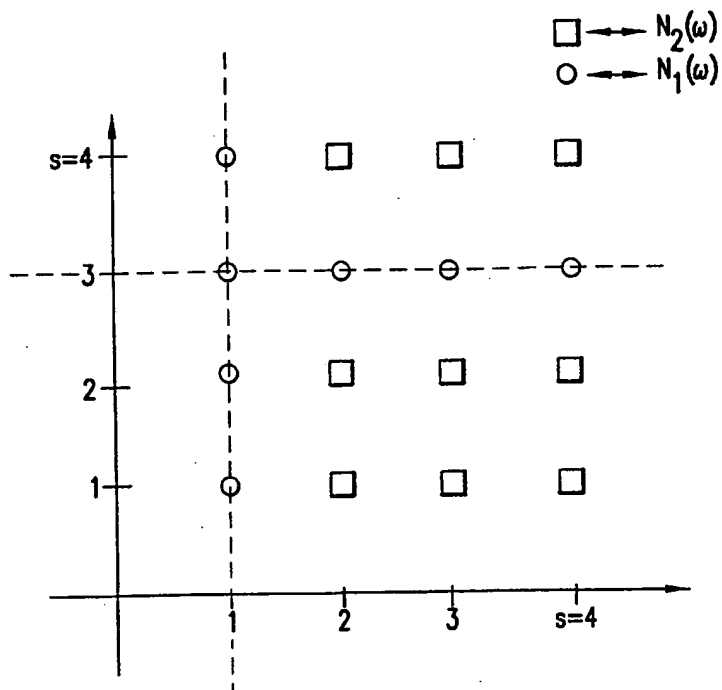
$N_1(\omega)=\{(1,1),(1,2),(1,4),(2,3),(3,3),(4,3)\}$
$N_2(\omega)=\{2,1),(3,1),(4,1),(2,2),(3,2),(4,2),(2,4),(3,4),(4,4)\}$
$\tilde{N}_\delta(\omega)=N_1(\omega)\cup N_2(\omega)$

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FIG. 3

s=3
n=2

VERTEX OR MODE OF Γ 410

EDGES OF Γ 420

n=2
s=3

TECHNOLOGICAL GRAPH Γ

FIG. 4

$$n=2$$
$$s=3$$
$$\delta=1$$
$$\ell(\omega_{t-1}) > \ell(\omega'_{t-1})$$
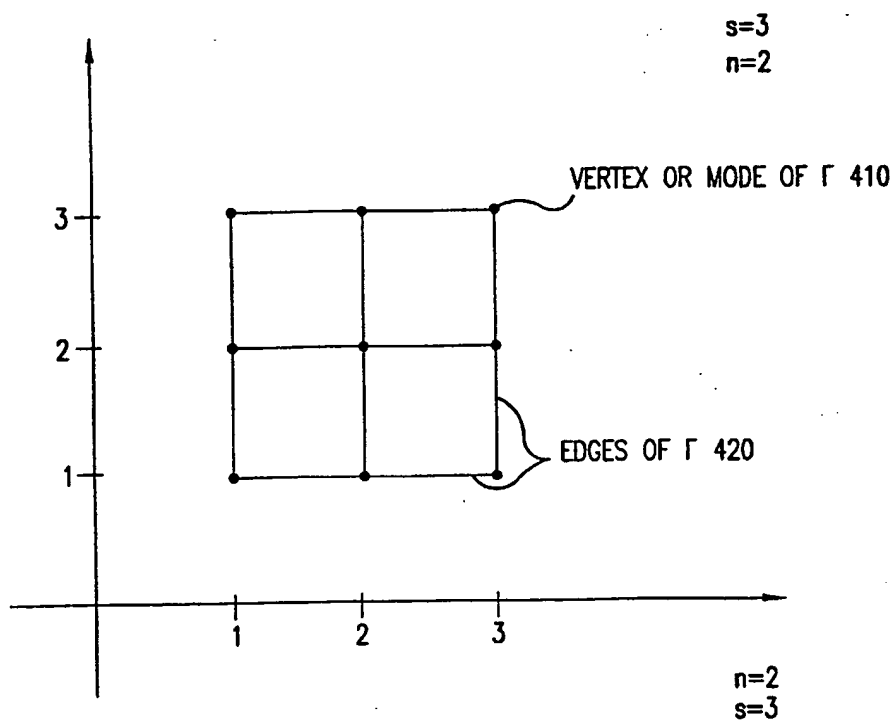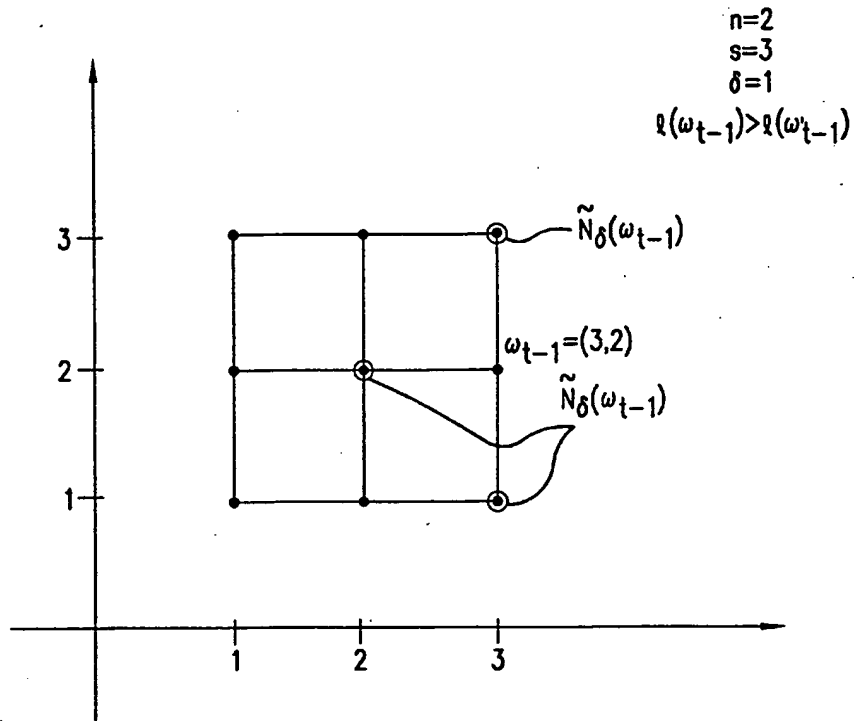
$$\tilde{N}_\delta(\omega_{t-1}) = \{(3,3),(3,1),(2,2)\}$$

$$\Rightarrow \# \tilde{N}_\delta(\omega_{t-1}) = 3$$

$$\Rightarrow \text{PROB } (\omega_{t-1}=\omega^*) = \begin{cases} 1/3, \omega^* \in \tilde{N}_\delta(\omega_{t-1}) \\ 0, \omega^* \notin \tilde{N}_\delta(\omega_{t-1}) \end{cases}$$

COST REDUCTION DYNAMICS

FIG. 5

TABLE 1: ESTIMATED PROGRESS RATIOS $\hat{p}$ (AND THE CORRESPONDING LEARNING COEFFICIENTS $\hat{b}$) FOR A VARIETY OF INDUSTRIES

INDUSTRY	$\hat{p}$ IN %	$\hat{b}$	#obs	R^2	t-STAT	COST MEASURE j	SOURCE
SEMICONDUCTORS	79	0.34[a]	42	0.77	-2.83	PRICE	GRUBER(1994)
SEMICONDUCTORS	75	0.41[a]	127	0.95	-27.92	PRICE	WEBBINK(1977)
SEMICONDUCTORS	80	0.32[a]	257	0.94	b	PRICE	IRWIN & KLENOW(1994)
NUCLEAR POWER[c]	73	0.46	41	NR[d]	-5.72	TOTAL COST	ZIMMERMAN(1982)
CHEMICAL PROCESSING	77	0.38	300	NR	-6.8	VALUE ADDED	LIEBERMAN(1987)
METAL PRODUCTS[e]	90	0.15	200	0.47	2.4	VALUE ADDED	DUDLEY(1972)
JUTE MILLS	94	0.09	243	0.82	f	VALUE-ADDED	KIBRIA & TISDELL(1985)
MUSICAL INSTRUMENTS	89	0.17	78	0.95	14.01	LABOR HOURS	BALOFF(1971)
APPAREL	77	0.37	33	0.94	9.4	LABOR HOURS	BALOFF(1971)
DIAMOND[g]	93	0.10	212	0.70	b	LABOR HOUR	LEVHARI & SHESHINSKI(1973)
MANUFACTURING[a,h]	89	0.17	438	0.75	b	VALUE-ADDED	SHESHINSKI(1967)
MANUFACTURING[i]	98	0.03	1,281	0.81	12	LABOR HOURS	BAHK & GORT(1993)
TRUCK ASSEMBLY	91	0.14	99	0.98	b	LABOR HOURS	EPPLE ET AL.(1991)

METHOD OF ESTIMATION IN ALL BUT ONE CASE IS ORDINARY LEAST SQUARES. THE ONE EXCEPTION, ZIMMERMAN (1982), EMPLOYED GENERALIZED LEAST SQUARES. THE ENTRY IN THE #obs COLUMN IS THE NUMBER OF DIFFERENT PRODUCTION OBSERVATIONS IN THE LEARNING CURVE

a: AVERAGE OF MORE THAN ONE ESTIMATED COEFFICIENT VALUE.
b: SIGNIFICANT COEFFICIENTS (SOURCE REPORTS THE STANDARD ERROR OF THE COEFFICIENT).
c: CONSTRUCTION OF NUCLEAR POWER PLANTS.
d: NR: NOT REPORTED.
e: MANUFACTURE OF METAL PRODUCTS.
f: SIGNIFICANT AT THE 1% LEVEL.
g: SAWING, CUTTING AND POLISHING.
h: DATA FROM 18 DIFFERENT TYPES OF PRODUCTS.
i: COMPRISING 41 DIFFERENT INDUSTRIES.
j: PER UNIT OF OUTPUT IN THE CASE OF "PRICE" (i.e. SELLING PRICE), "LABOR HOURS" AND "TOTAL COST"; PER WORKER IN THE CASE OF "VALUE ADDED."

FIG. 6

PARAMETER	DESCRIPTION	RANGE
n	NUMBER OF OPERATIONS PER RECIPE	POSITIVE INTEGERS
s	NUMBER OF SETTINGS PER OPERATION	POSITIVE INTEGERS
e	NUMBER OF INTRAFIRM EXTERNALITIES PER OPERATION	$\{1,...,n\}$
B	BATCH SIZE	1
τ	NUMBER OF TRIALS PER MEASURED BATCH	POSITIVE INTEGERS
δ	NUMBER OF OPERATIONS ALTERED PER TRIAL	$\{1,...,n\}$
T	LENGTH OF THE PRODUCTION RUN	POSITIVE INTEGERS

TABLE 2: SUMMARY OF PARAMETERS

FIG.7

	MINIMUM	MAXIMUM	MEAN	STD. DEVIATION
PARAMETERS				
n (# OPS.)	1.0000	100.00	179.855	308.577
s (SET./OP.)	2.0000	100.00	15.8150	24.3057
e (EXT. PAR.)	1.0000	100.00	9.653179	21.5503
δ (TRIAL DIST.)	1.0000	100.00	11.9653	27.0370
T (LENGTH OF RUN)	10.0000	1,000.00	704.740	371.619
RESULTS				
$\bar{p}$ (MEAN PROG.)	60.8610	100.00	89.8573	10.7914
s_p (ST. DEV. PROG.)	.0000	23.0056	2.407138	3.221978
$\bar{z}$ (MEAN IMP. PERCENT)	.0000	98.4000	24.1670	27.7214
$\bar{c}_2$ (MEAN CURV.)	-.1315	.1634	.0040	.042
s_{c2} (ST. DEV. CURV.)	.0000	.4245	.025	.047

TABLE 3: SUMMARY STATISTICS: THE FOCUSED PARAMETER SET (173 PARAMETER VECTORS; 20 RUNS PER PARAMETER VECTOR)

FIG.8

PARAMETERS	MINIMUM	MAXIMUM	MEAN	STD. DEVIATION
n (#OPS.)	2.0000	999.00	499.3160	288.9405
s (SET./OP.)	2.0000	99.00	50.8000	28.4154
e (EXT. PAR.)	1.0000	10.00	5.4000	2.8725
δ (TRIAL DIST.)	1.0000	10.00	5.4880	2.8653
T (LENGTH OF RUN)	1,000.00	1,000.00	1,000.00	.0000
RESULTS				
$\bar{p}$ (MEAN PROG.)	74.36	95.72	91.6939	3.4291
s_p (ST. DEV. PROG.)	.30	9.81	8.9121	5.5263
$\bar{z}$ (MEAN IMP. PERCENT)	.66	36.24	8.9121	5.5263
s_z (ST. DEV. PERCENT OBS. IMP.)	.13	1.79	.5825	.2580
$\bar{c}_2$ (MEAN QUAD. COEF.)	-.07	.09	-.017	.0128
s_{c2} (STD. DEV. QUAD. COEF.)	.00	.07	.00613	.00750
l_o (MEAN INIT. LABOR REQ.)	.35	.52	.4965	.0126
s_{lo} (STD. DEV. INIT. LABOR REQ.)	.01	.16	.0172	.0158
l_T (MEAN TERM. LABOR REQ.)	.05	.76	.6052	.1190
s_{lT} (STD. DEV. TERM. LABOR REQ.)	-.07	.09	.017	.0128

TABLE 4: SUMMARY STATISTICS: THE RANDOM PARAMETER SET (250 PARAMETER VECTORS; 20 RUNS PER PARAMETER VECTOR)

FIG. 9

SUMMARY

R	R^2	ADJUSTED R^2	STD. ERROR OF THE ESTIMATE
.836	.698	.689	6.015016

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	13988.2	5	2797.635	22.325	.000
RESIDUAL	6042.130	167	36.180		
TOTAL	20030.3	172			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	101.537	1.798		56.476	.000
LOG n	-1.22	.276	-.021	-.443	.000
LOG s	-7.115	.469	-.756	-15.157	.000
LOG e	3.132	.361	.397	8.672	.000
LOG δ	.343	.353	.049	.970	.333
LOG τ	-.064	.250	-.011	-.256	.798

TABLE 5: PREDICTION OF $\bar{p}$ BY OLS SCORING FROM THE FOCUSED PARAMETER SET

FIG. 10

SUMMARY

R	R^2	ADJUSTED R^2	STD. ERROR OF THE ESTIMATE
.917	.840	.837	1.3847

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	2460.072	5	492.014	256.606	.000
RESIDUAL	467.843	244	1.917		
TOTAL	2927.914	249			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	74.119	.699		106.037	.000
LOG n	2.118	.090	.620	23.586	.000
LOG s	-.128	.103	-.033	-1.246	.214
LOG θ	2.341	.125	.484	18.750	.000
LOG δ	1.493	.129	.305	11.563	.000
LOG τ	-.127	.123	-.027	-1.037	.301

TABLE 6: PREDICTION OF $\bar{p}$ BY OLS SCORING FROM THE RANDOM PARAMETER SET

FIG. 11

SUMMARY

R	R²	ADJUSTED R²	STD. ERROR OF THE ESTIMATE
.740	.547	.536	2.193557

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	977.192	4	244.298	50.772	.000
RESIDUAL	808.364	168	4.812		
TOTAL	1785.557	172			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	2.656	0.637		4.168	0.000
LOG n	-0.740	0.100	-0.417	-7.366	0.000
LOG s	1.403	0.171	0.499	8.201	0.000
LOG e	-.345	0.132	-0.146	-2.624	0.009
LOG δ	.219	.128	.104	1.709	0.089

TABLE 7: PREDICTION OF s_ρ BY OLS SCORING FROM THE FOCUSED PARAMETER SET

FIG.12

SUMMARY

R	R^2	ADJUSTED R^2	STD. ERROR OF THE ESTIMATE
.831	.691	.685	.5226

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	149.101	5	29.820	109.186	.000
RESIDUAL	66.640	244	.273		
TOTAL	215.741	249			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	5.704	.264		21.620	.000
LOG n	-.722	.034	-.779	-21.313	.000
LOG s	-.023	.039	-.022	-.601	.549
LOG e	-.137	.047	-.105	-2.915	.004
LOG δ	-.121	.049	-.091	-2.485	.014
LOG τ	-.103	.046	-.080	-2.215	.028

TABLE 8: PREDICTION OF S_p BY OLS SCORING FROM THE RANDOM PARAMETER SET

FIG. 13

SUMMARY

R	R^2	ADJUSTED R^2	STD. ERROR OF THE ESTIMATE
.772	.596	.595	3.5315

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	91718.6	4	22929.7	1838.589	.000
RESIDUAL	62294.3	4995	12.471		
TOTAL	154013	4999			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	-1.786	.396		-4.511	.000
LOG n	3.240	.050	.585	64.163	.000
LOG s	.213	.059	.033	3.630	.549
LOG e	-3.954	.071	-.504	-55.604	.004
LOG δ	-2.221	.074	-.280	-30.163	.000

TABLE 9: PREDICTION OF $\bar{z}$ BY OLS SCORING FROM THE RANDOM PARAMETER SET

FIG. 14

SUMMARY

R	R^2	ADJUSTED R^2	STD. ERROR OF THE ESTIMATE
.602	.362	.361	.0127

ANALYSIS OF VARIANCE

	SUM OF SQUARES	df	MEAN SQUARE	F	SIG.
REGRESSION	.457	5	.0914	566.462	.000
RESIDUAL	.806	4994	.000161		
TOTAL	1.262	4999			

COEFFICIENTS

	COEFFICIENT	STD. ERROR	STD. COEFFICIENT	t	SIG.
CONSTANT	-.0071	.001		-4.919	.000
LOG n	-.0050	.000	-.314	-27.020	.000
LOG s	-.00072	.000	-.040	-3.422	.001
LOG e	.0105	.000	.466	40.872	.000
LOG δ	-.00592	.000	.260	22.345	.000
LOG τ	-.0016	.000	-.072	-6.281	.000

TABLE 10: PREDICTION OF $\bar{c}_2$ BY OLS SCORING FROM THE RANDOM PARAMETER SET

FIG. 15

FIG.16

FIG. 17
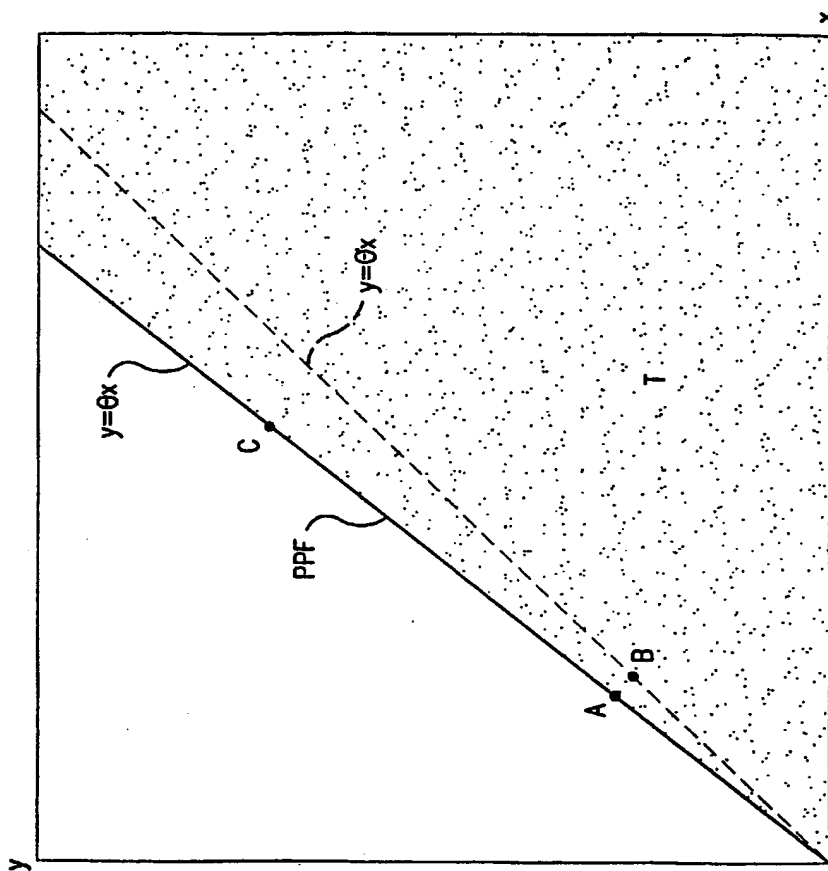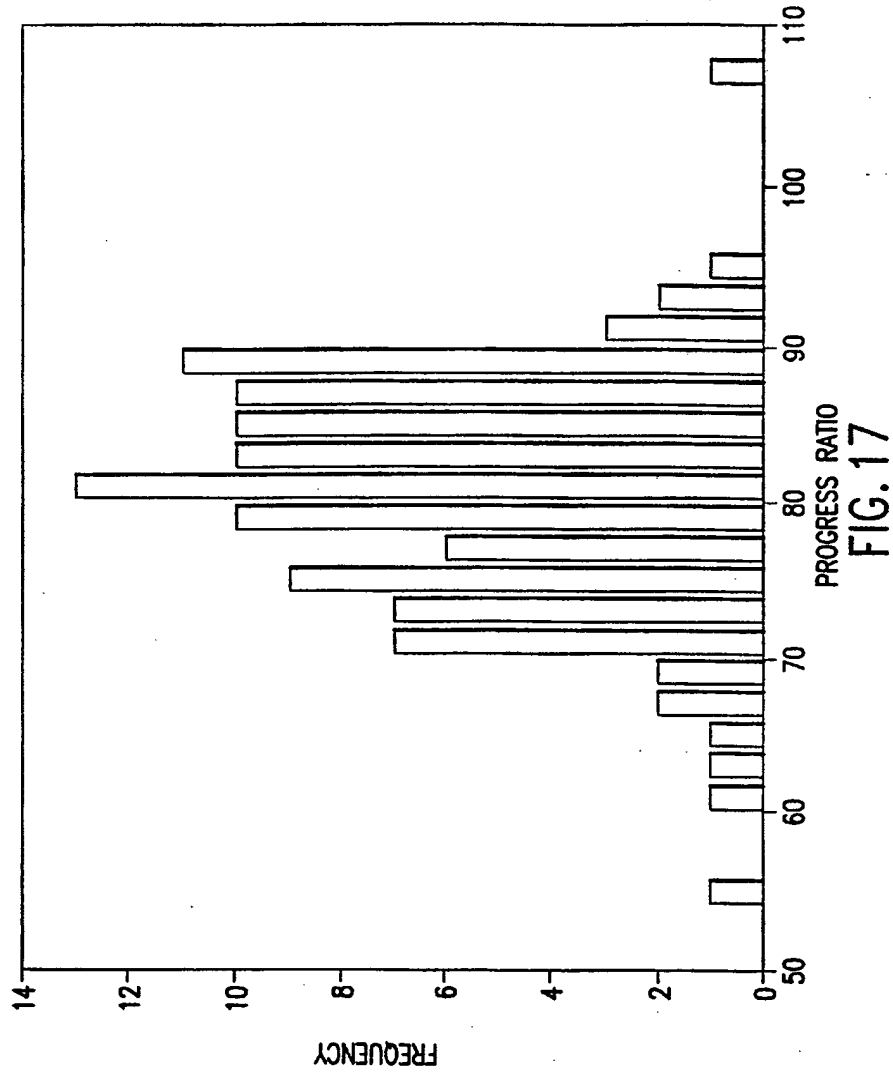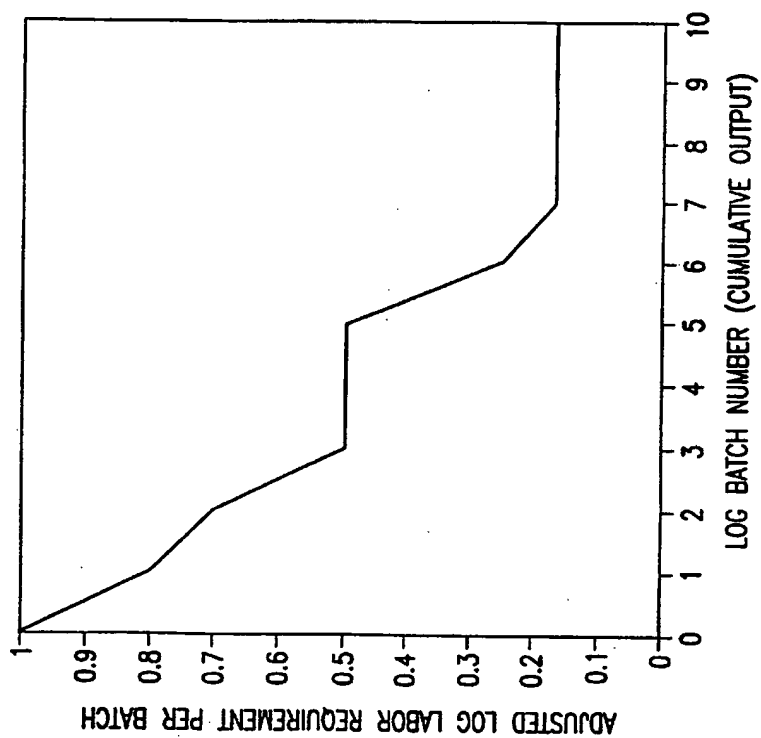
FIG. 18
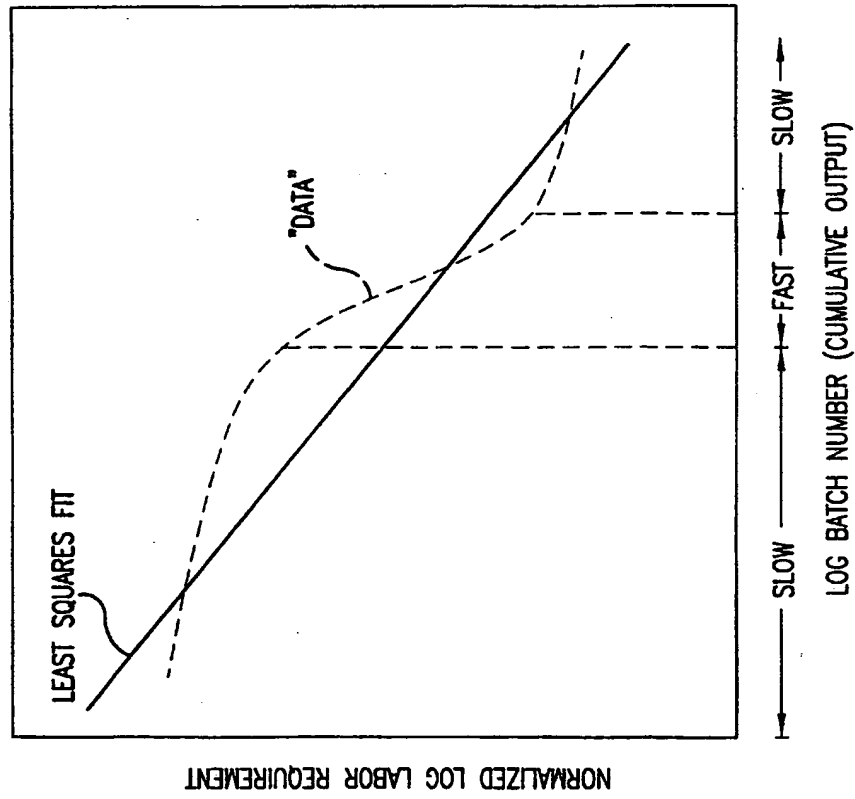
FIG. 19
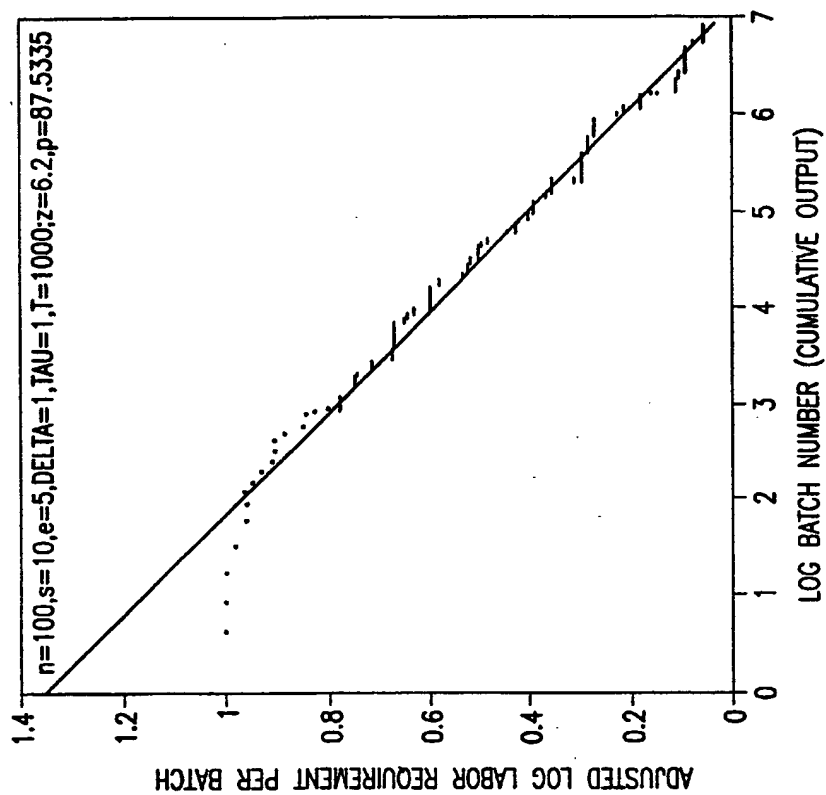
FIG.20
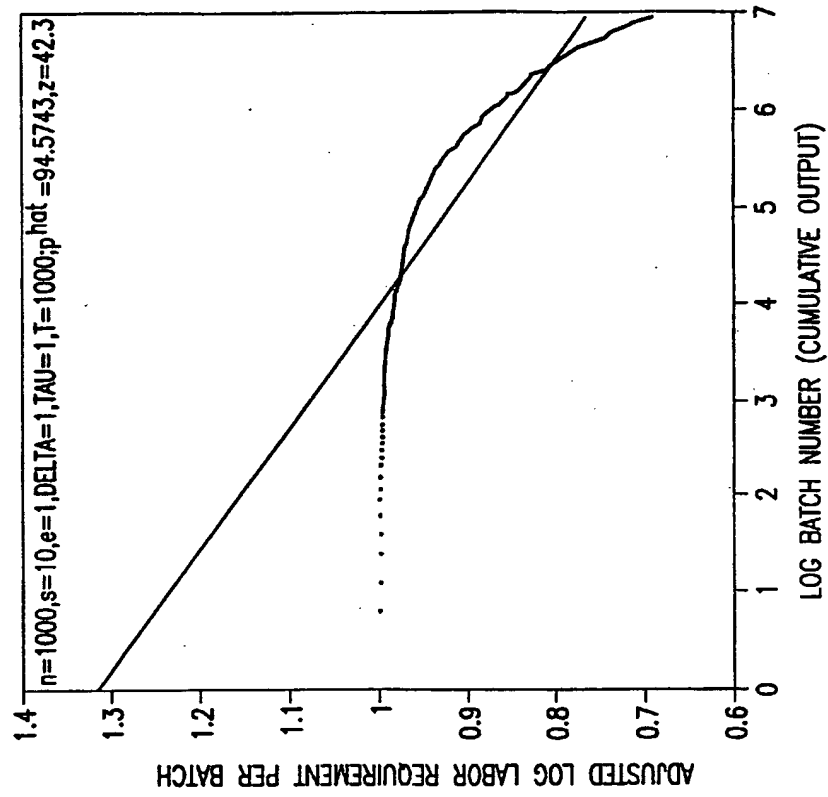
FIG.21
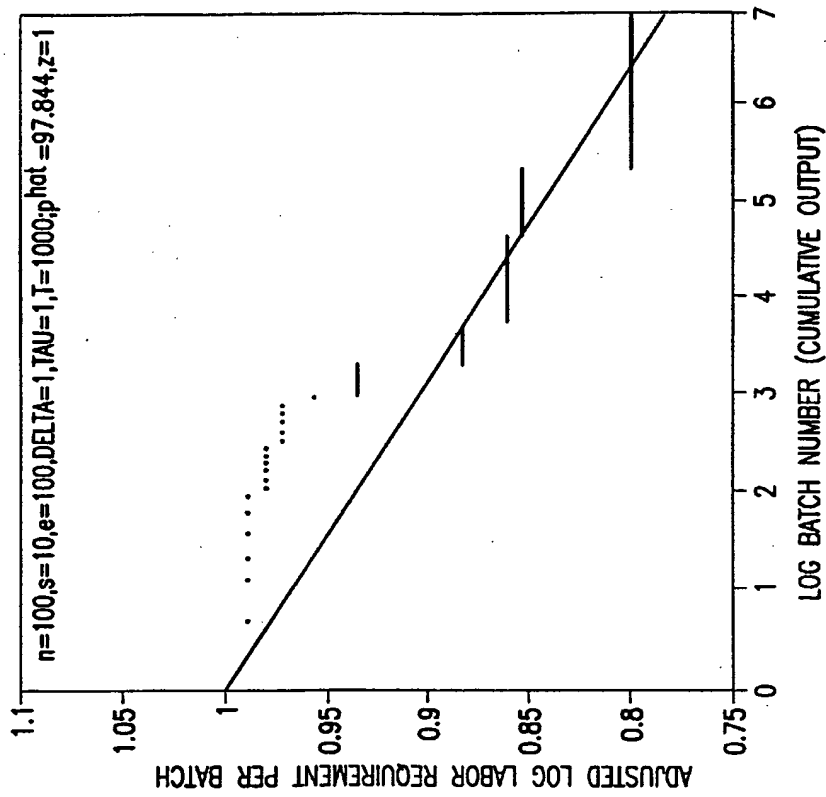
FIG.22
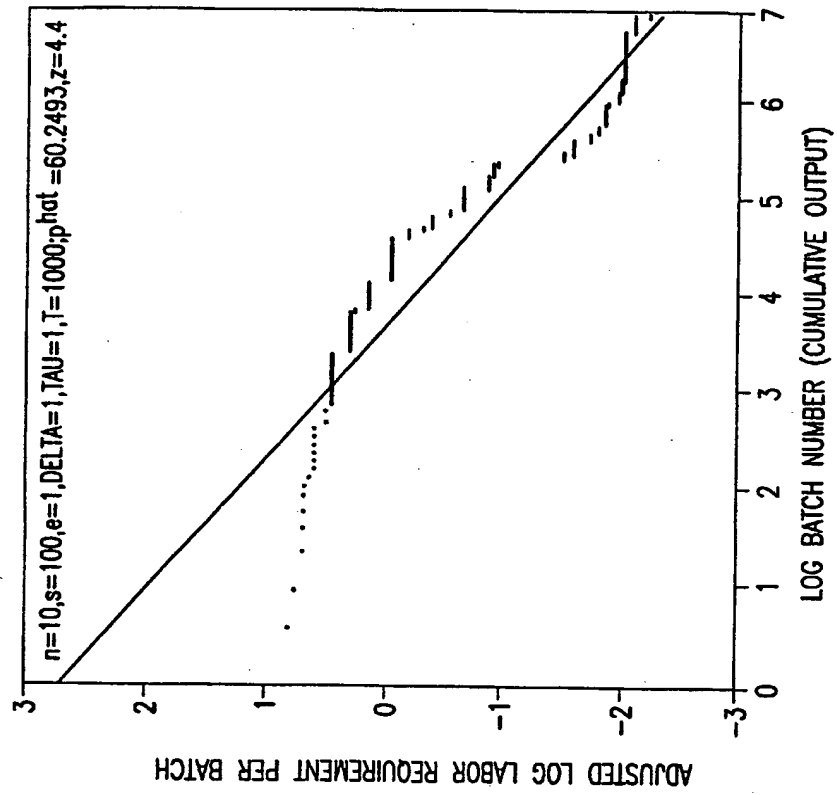
FIG.23
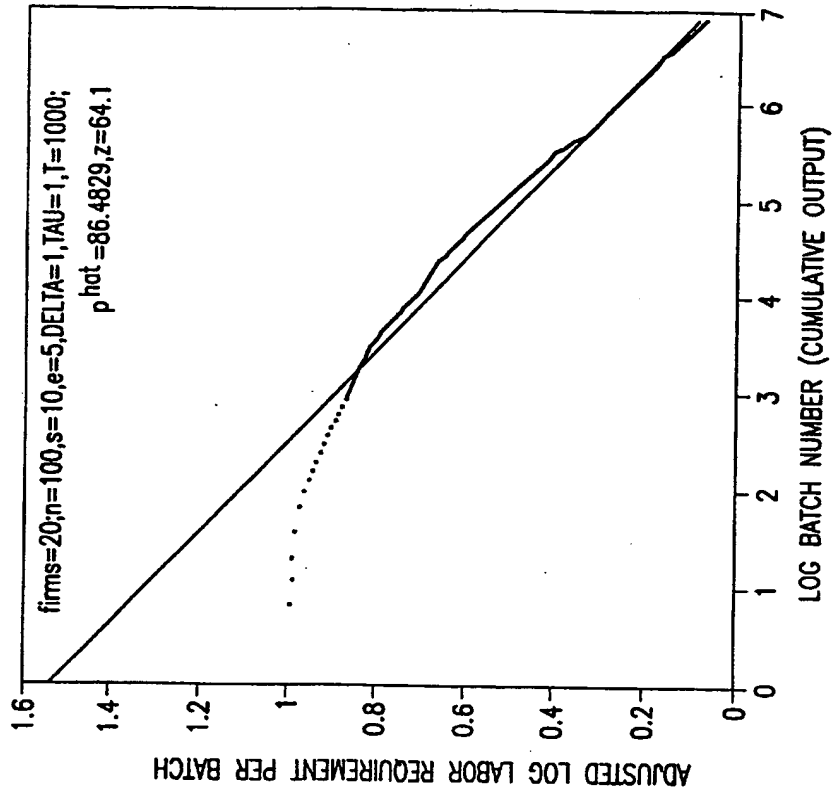
FIG.24

FIG.25

FIG.26

firms=50;n=100,s=10,e=100,DELTA=1,TAU=1,T=1000;
$p^{hat}=98.4537, z=15.7$

LOG BATCH NUMBER (CUMULATIVE OUTPUT)

ADJUSTED LOG LABOR REQUIREMENT PER BATCH

FIG.27

firms=20;n=10,s=100,e=1,DELTA=1,TAU=1,T=1000;
p^{hat}=60.5439,z=57.7

ADJUSTED LOG LABOR REQUIREMENT PER BATCH

LOG BATCH NUMBER (CUMULATIVE OUTPUT)

FIG.28

FIG. 29
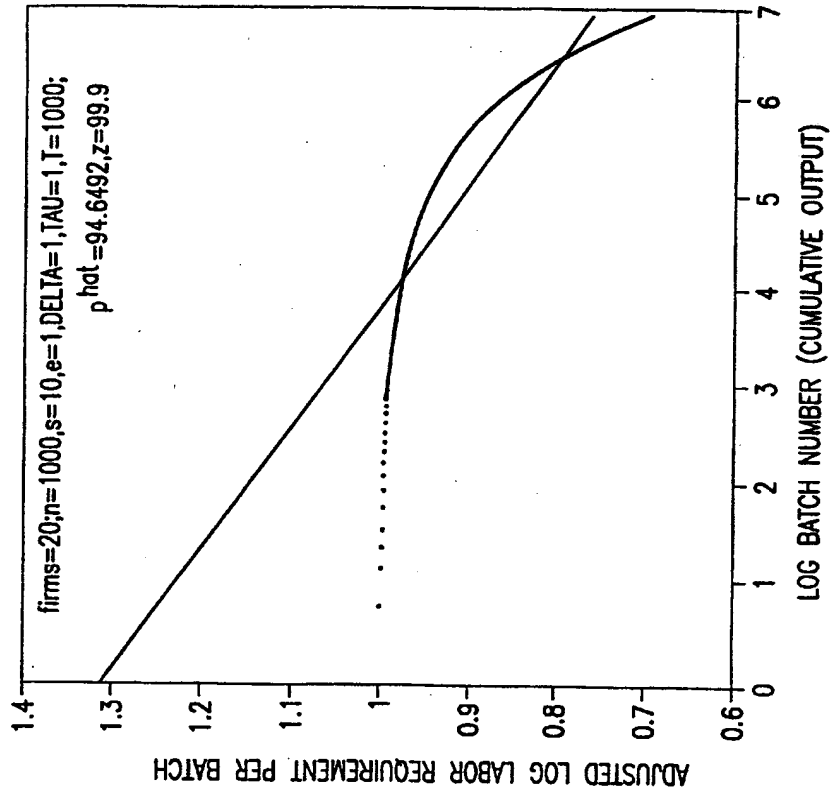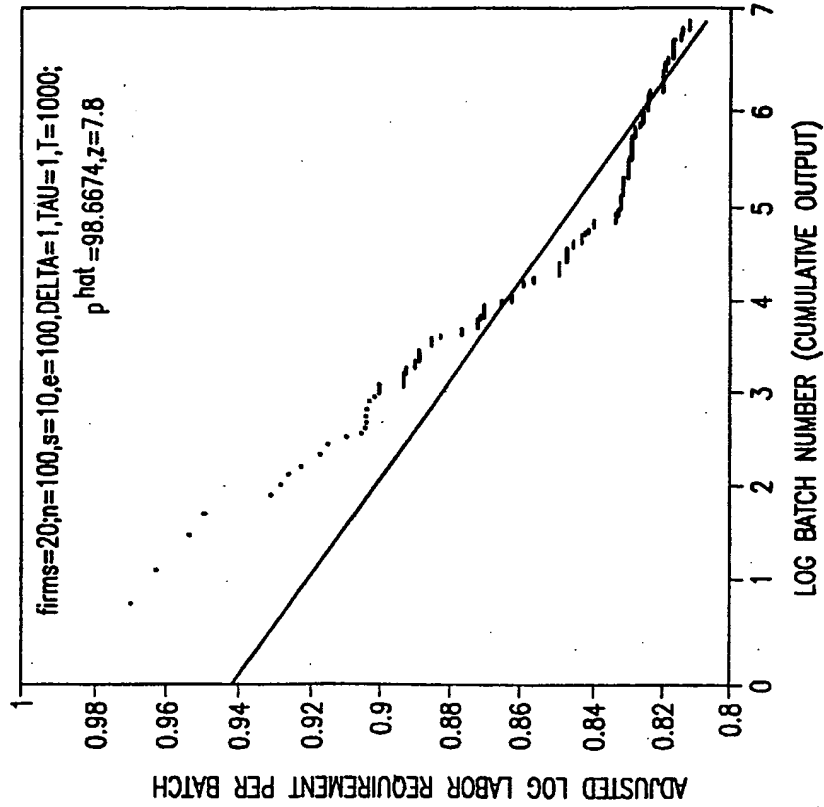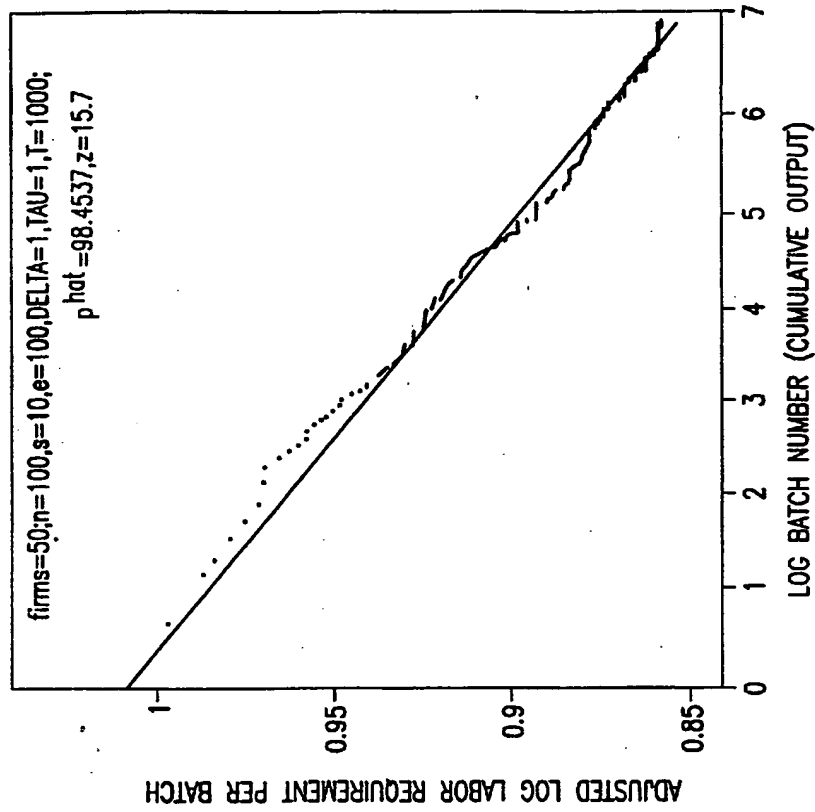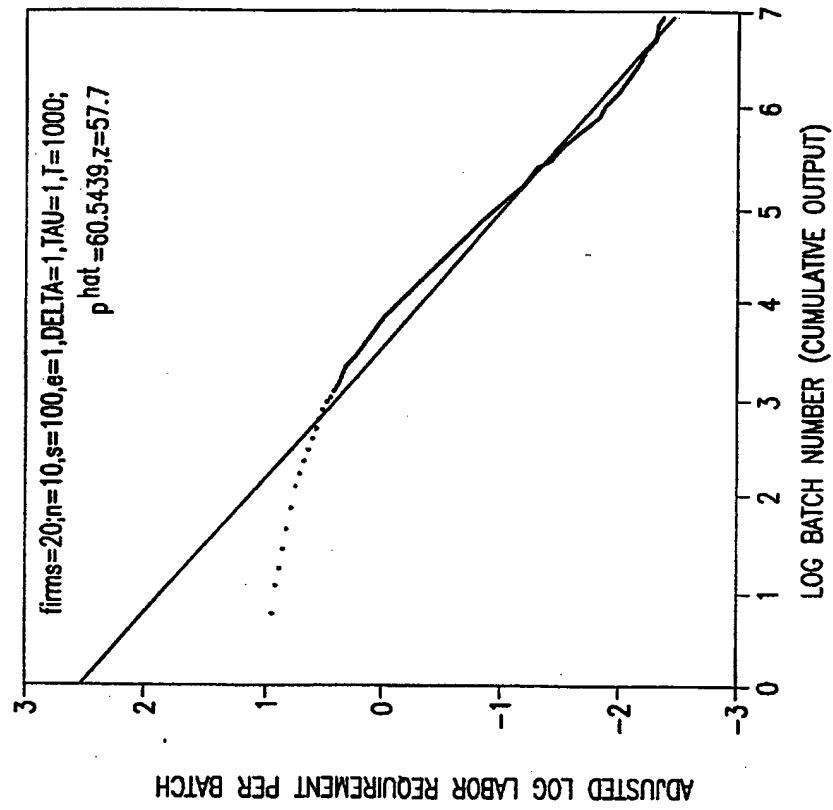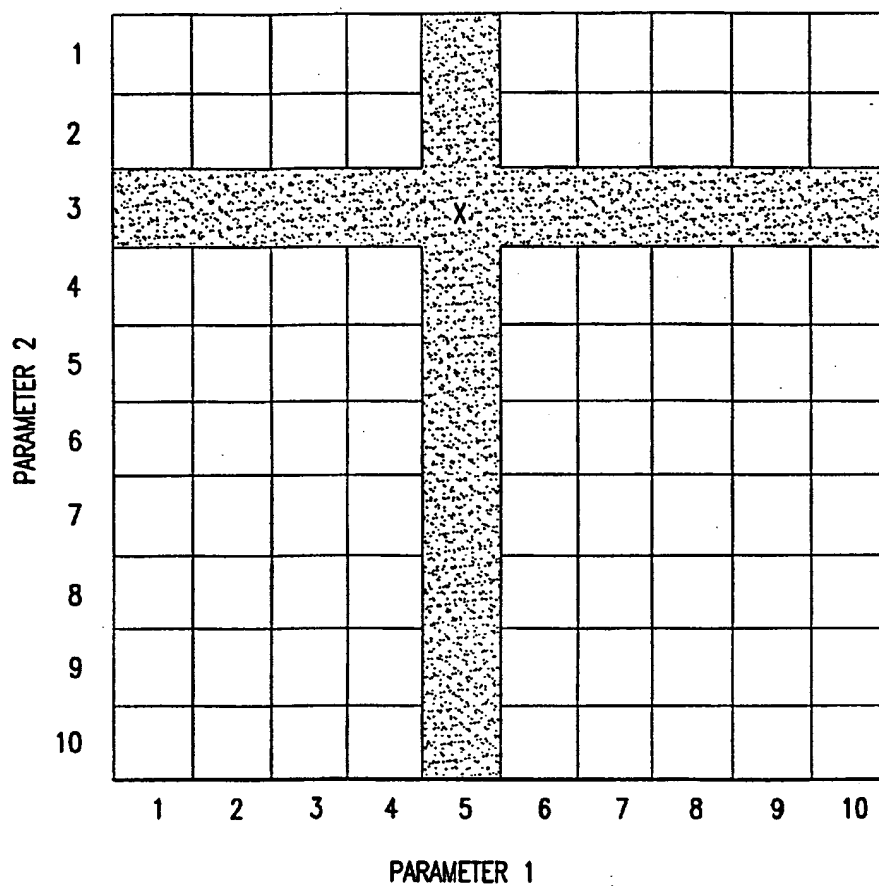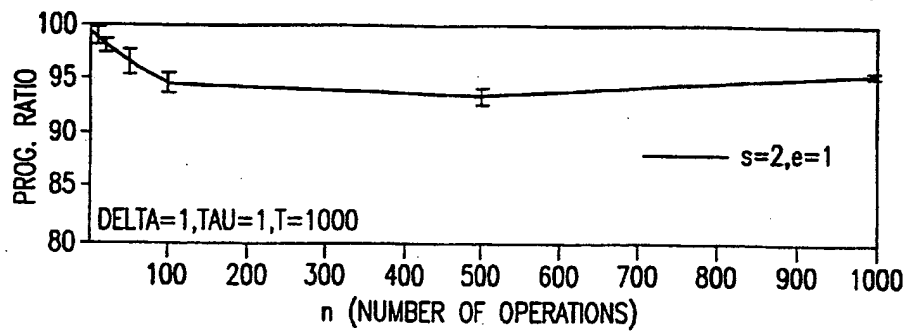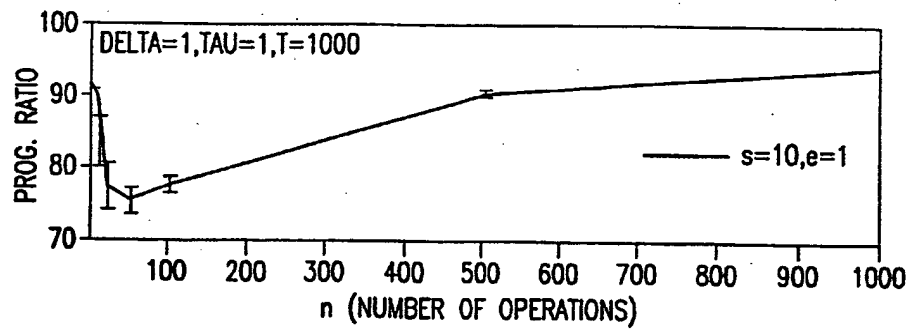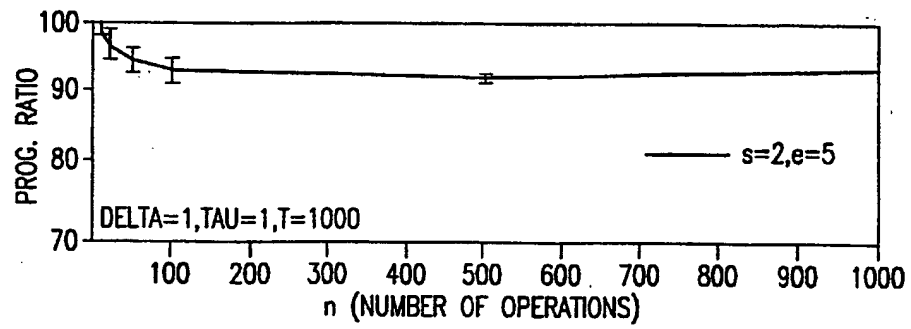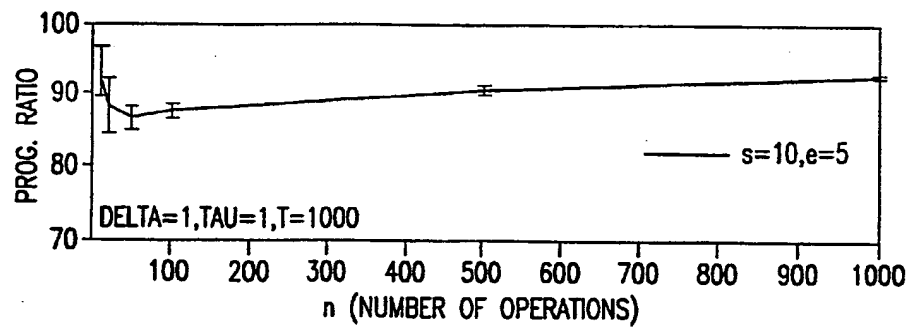
FIG.30A

FIG.30B

FIG.30C

FIG.30D
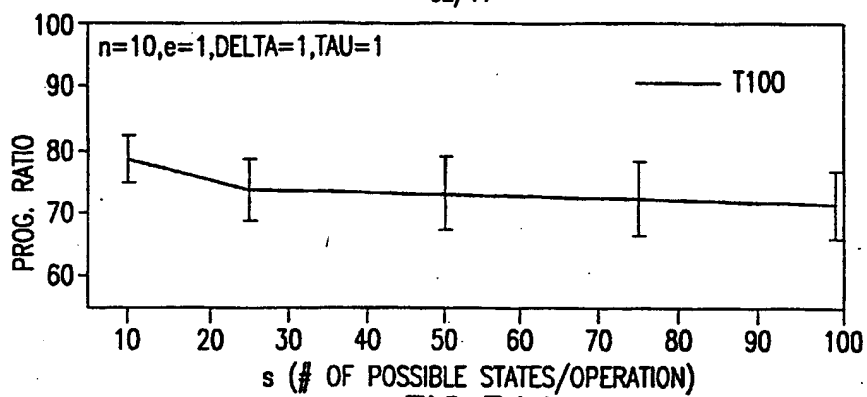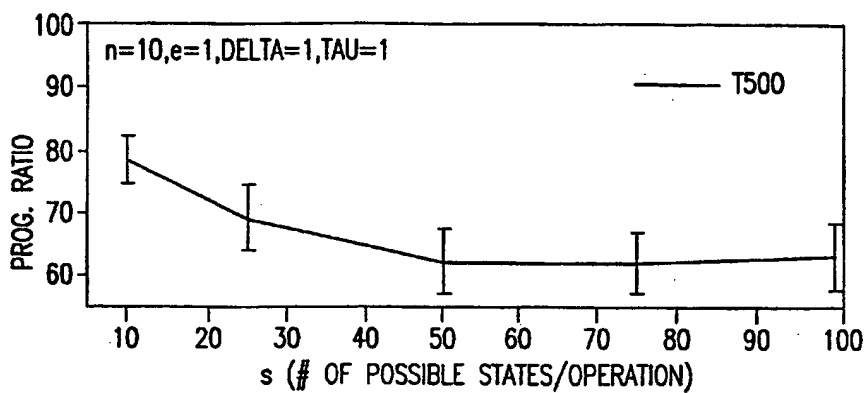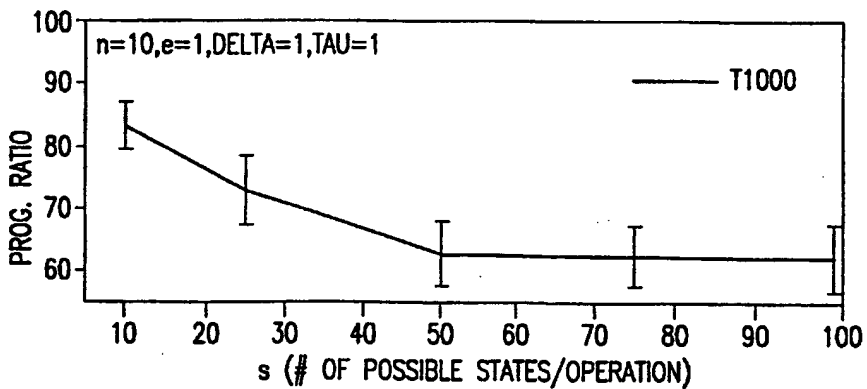
FIG.31A

FIG.31B

FIG.31C
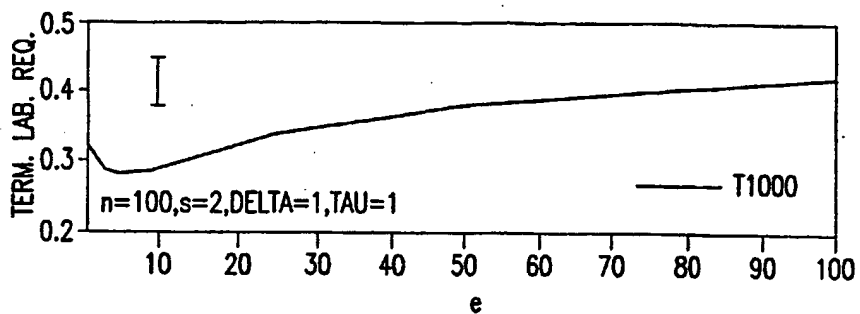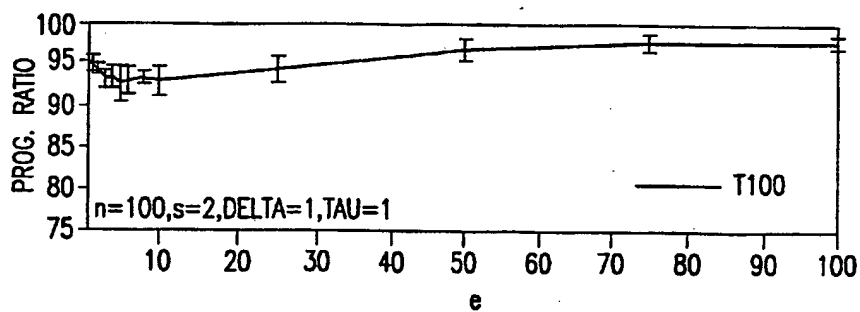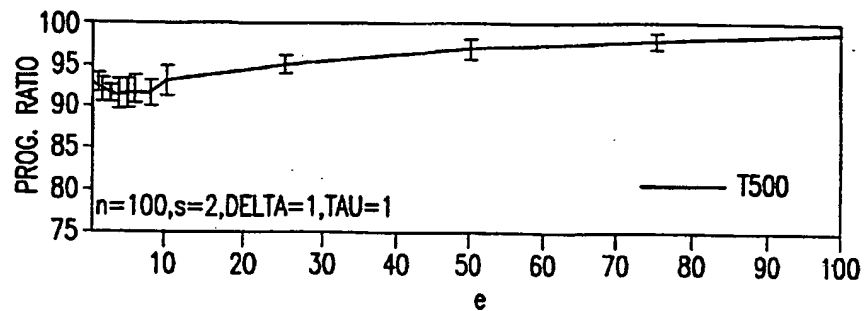
FIG. 32A
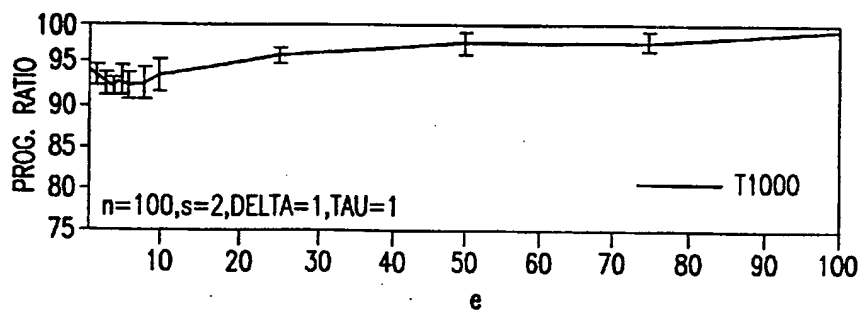
FIG. 32B

FIG. 32C

FIG. 32D
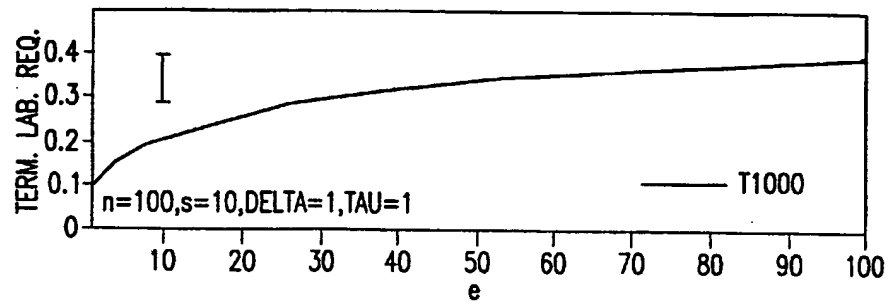
FIG.33A
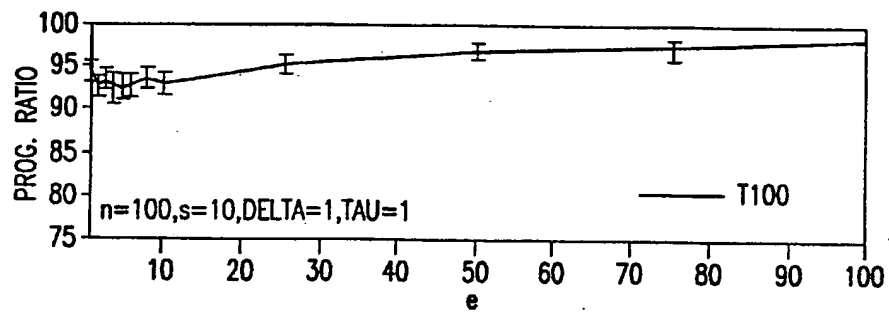
FIG.33B
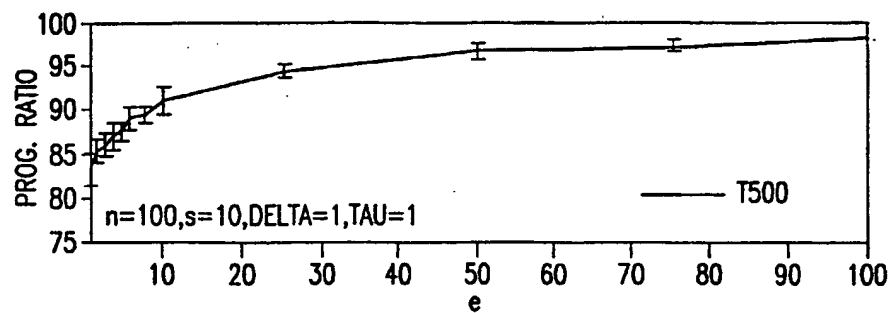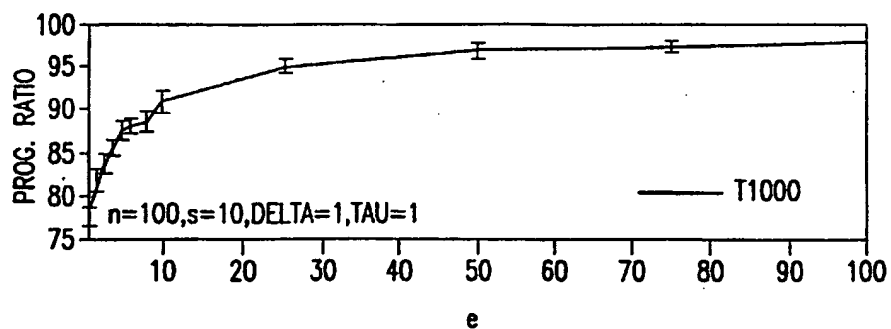
FIG.33C

FIG.33D
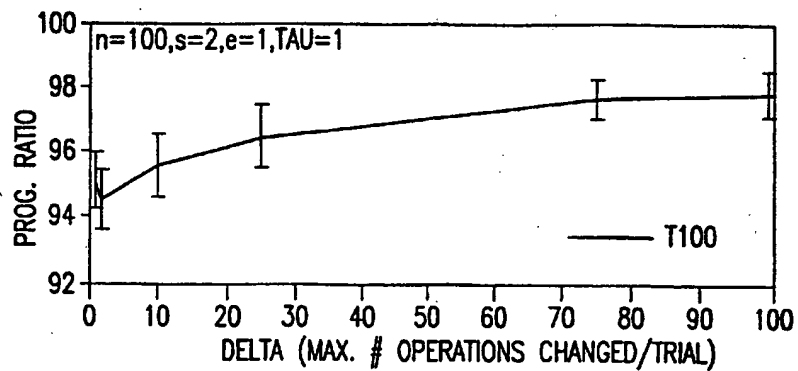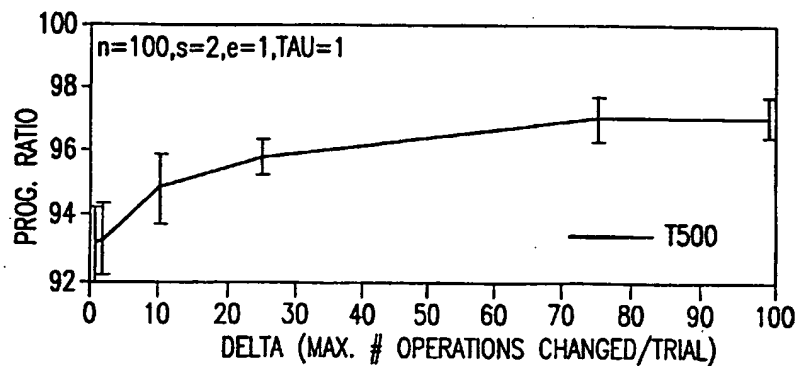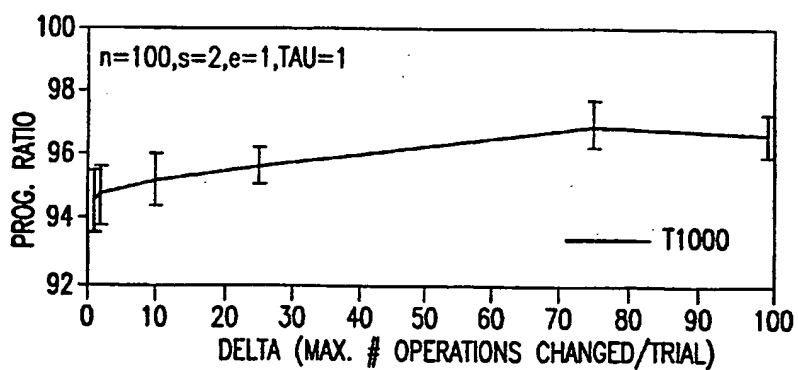
FIG.34A

FIG.34B

FIG.34C
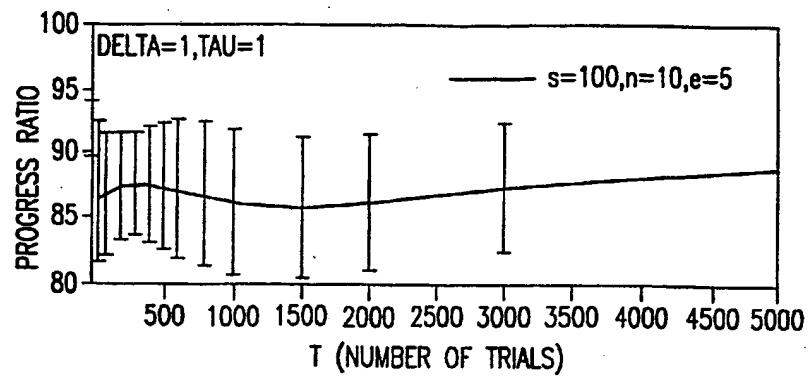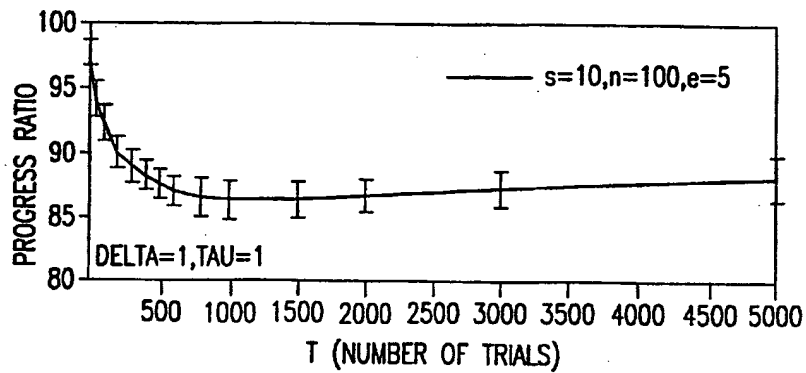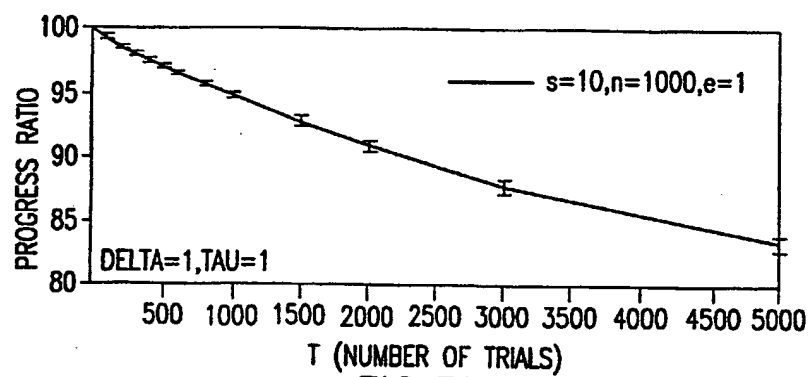
FIG.35A

FIG.35B

FIG.35C
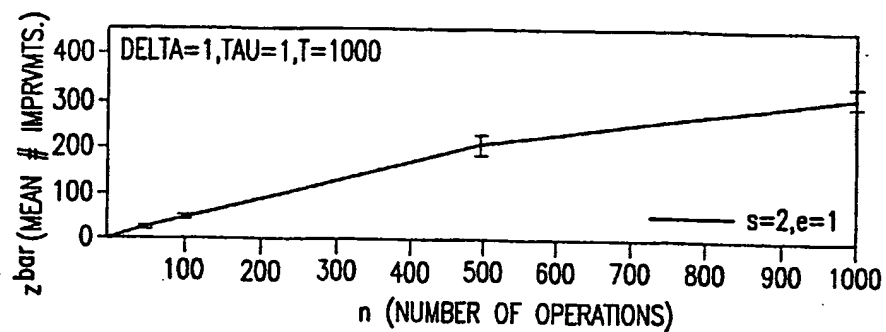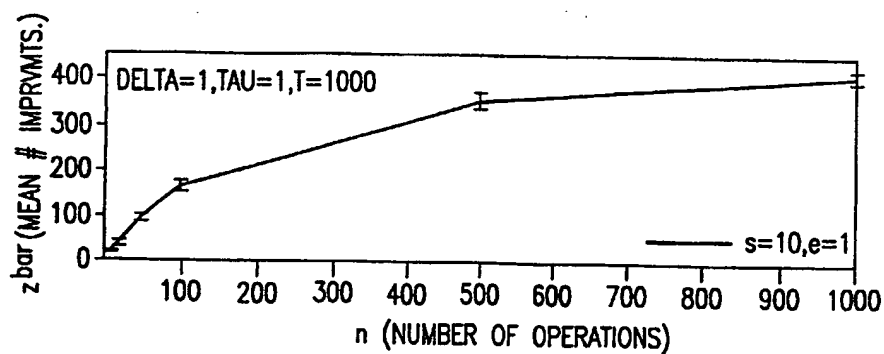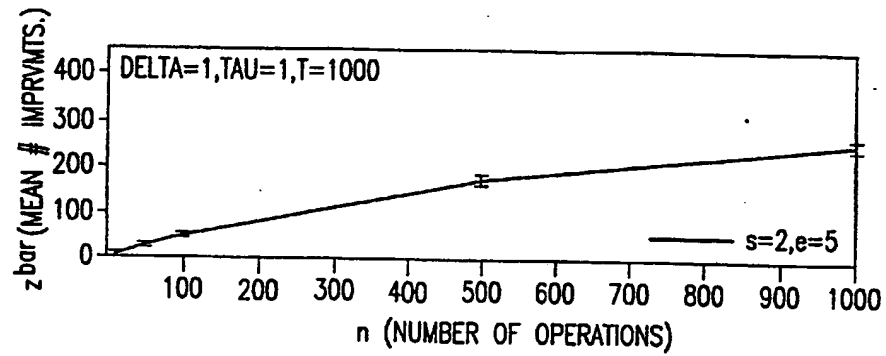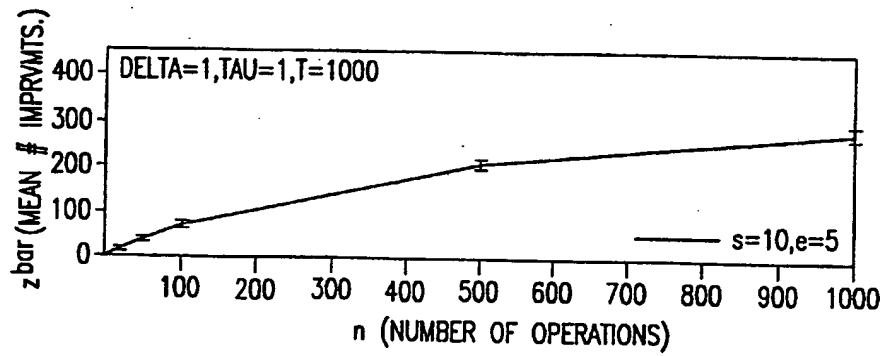
FIG.36A

FIG.36B

FIG.36C

FIG.36D
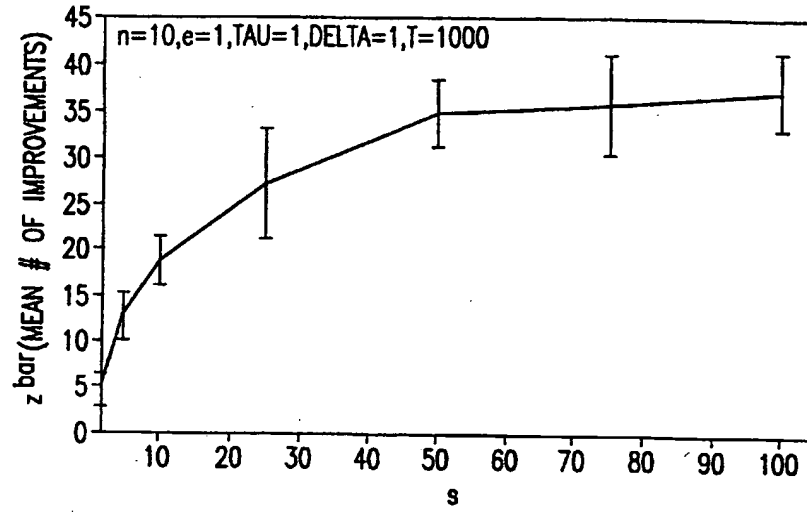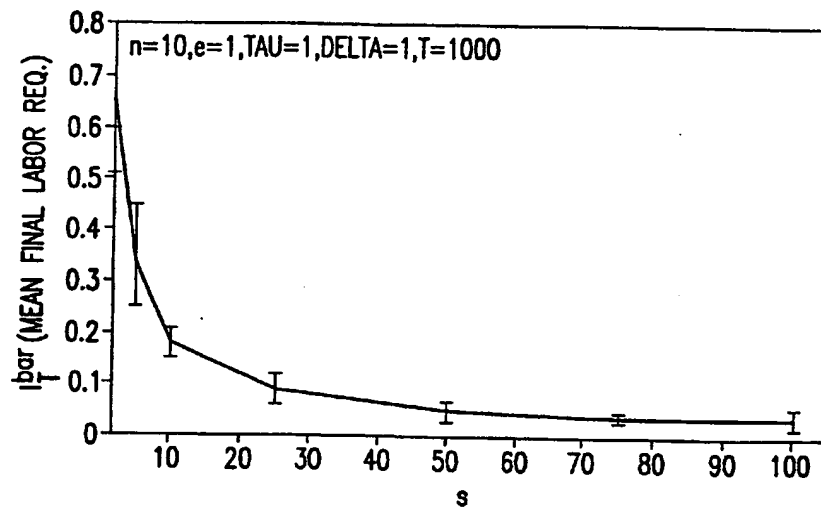
FIG.37A

FIG.37B
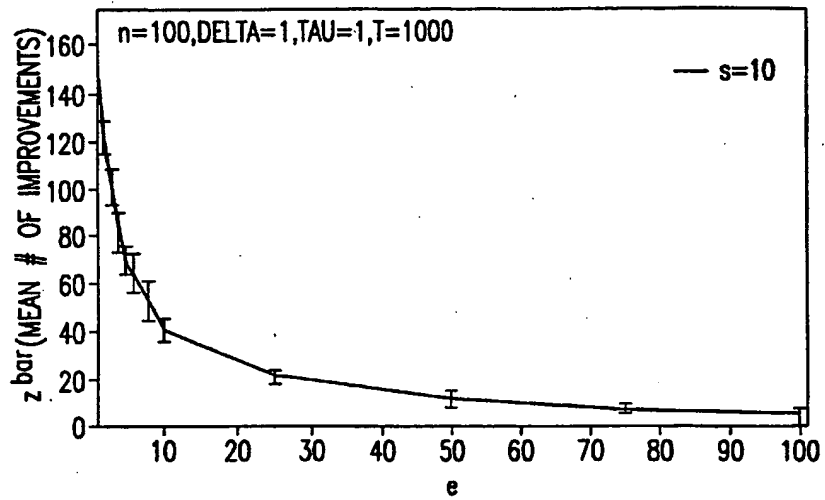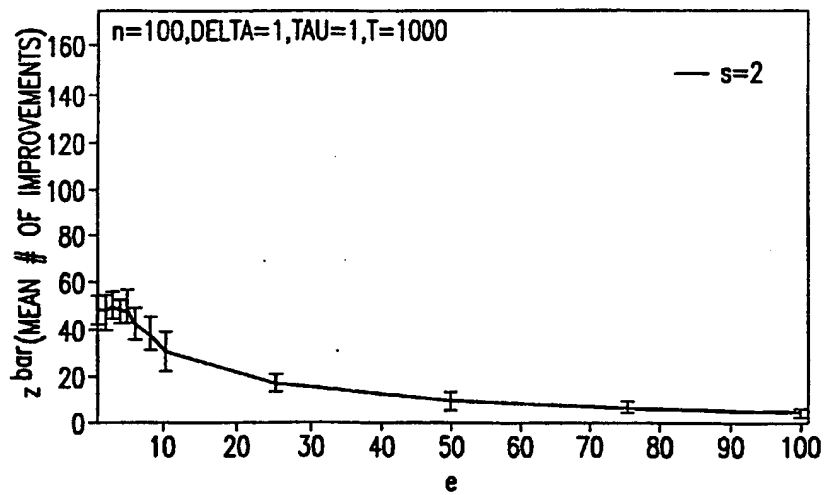
FIG.38A

FIG.38B
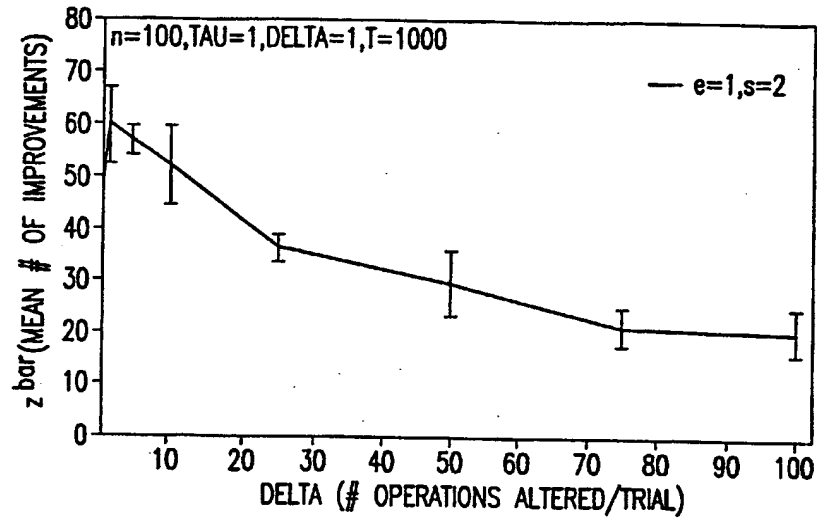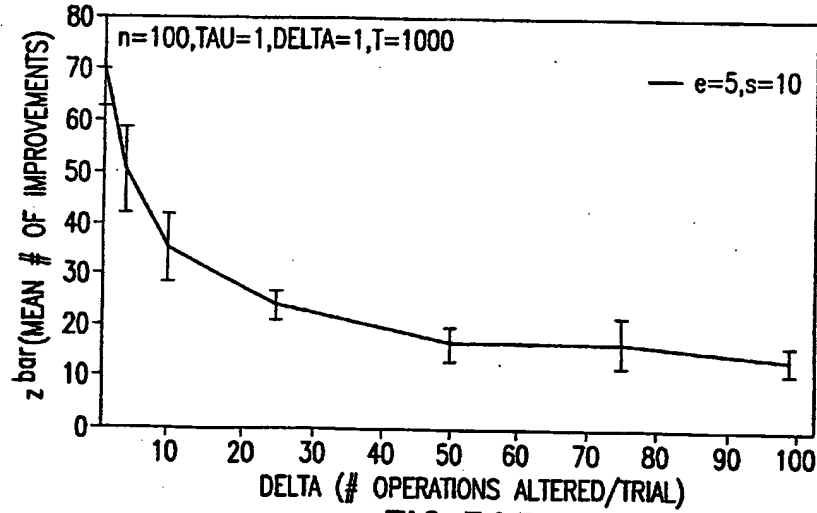
FIG.39A

FIG.39B
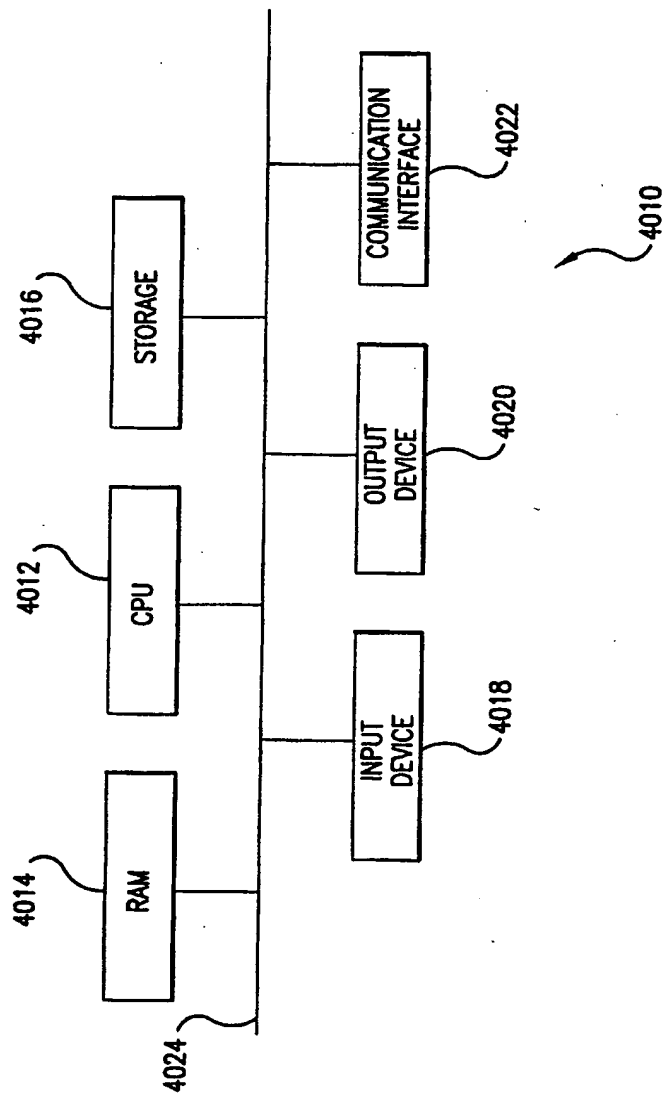
FIG. 40

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER

IPC(6)   :G06F 15/21,17/60
US CL   :705/1, 7, 8, 9

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

U.S. :   705/1, 7, 8, 9

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EAST (USPAT: JPO: EPO) : TRIAL AND ERROR: RESEARCH AND DEVELOPMENT: PRODUCTION: VECTOR: DISTANCE: OPTIMIZE: IMPROVE: COST

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 4,942,527 A (SCHUMACHER) 17 JULY 1990, col 2, lines 37-49; col 5, lines 38-52, 62-68; col 6, lines 1-9; col 14, lines 23-50; col 15, lines 5-68; col 16, lines 1-40	1-11, 20, 26-38, 41-48
Y	US 5,172,313 A (SCHUMACHER) 15 DECEMBER 1992, col 3, lines 50-55; col 5, lines 45-68; col 6, lines 1-19; col 21, lines 5-68	1-11, 20, 26-38, 41-48
Y	US 5,487,133 A (PARK et al) 23 JANUARY 1996, col 25, lines 15-43; col 28, lines 3-32	1-11, 20, 26-38, 41-48
Y	US 5,799,286 A (MORGAN et al) 25 AUGUST 1998, col 1, lines 10-56; col 5, lines 13-34; col 6, lines 52-67; col 8, lines 32-52; col 21, lines 1-15	1-11, 20, 26-38, 41-48

[X] Further documents are listed in the continuation of Box C.   [ ] See patent family annex.

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"&"   document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
14 JANUARY 2000	1 0 FEB 2000

Name and mailing address of the ISA/US	Authorized officer
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Form PCT/ISA/210 (second sheet)(July 1992)*

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT		
Category*	Citation of document, with indication. where appropriate. of the relevant passages	Relevant to claim No.
Y	JP 07334576 (HAYASHI et al) 22 DECEMBER 1995, constitution (english translation)	1-11, 20, 26-38, 41-48